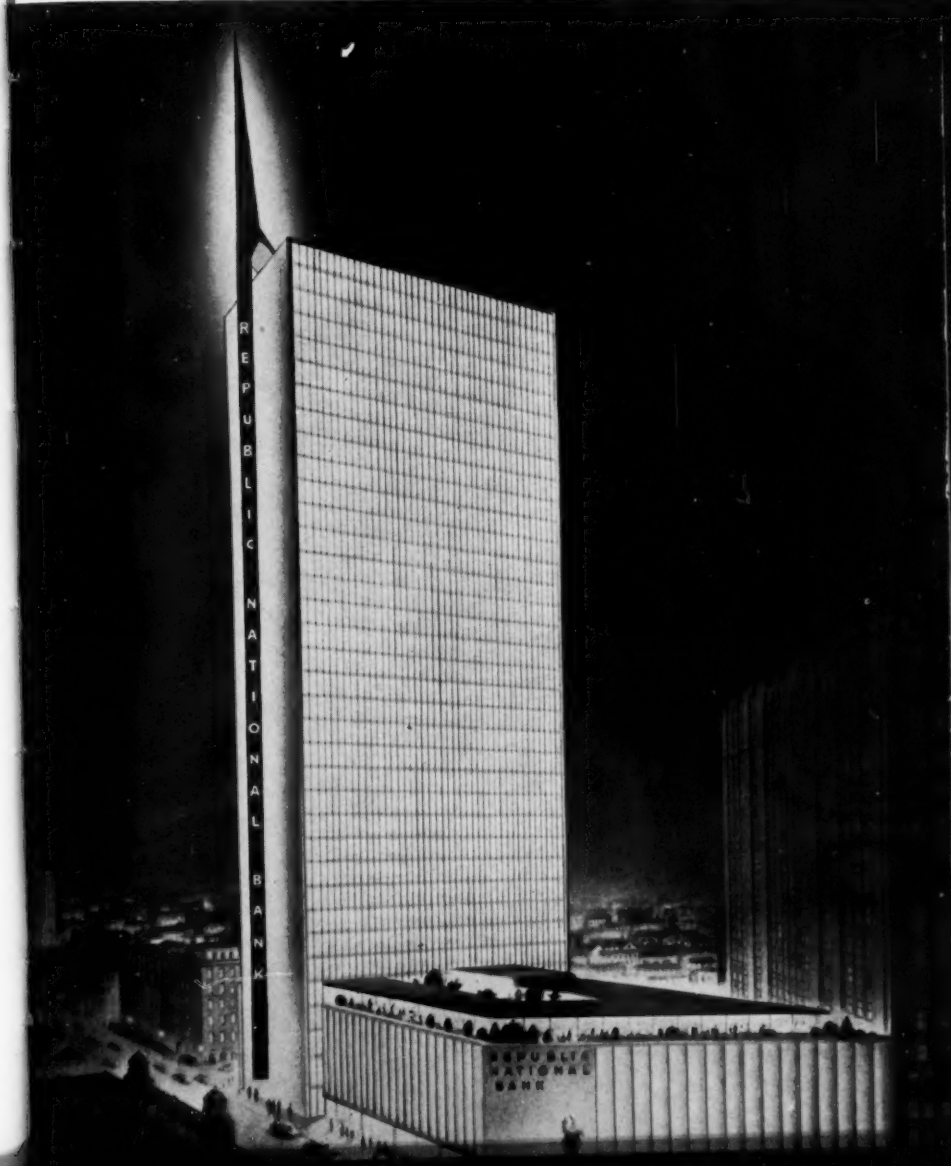
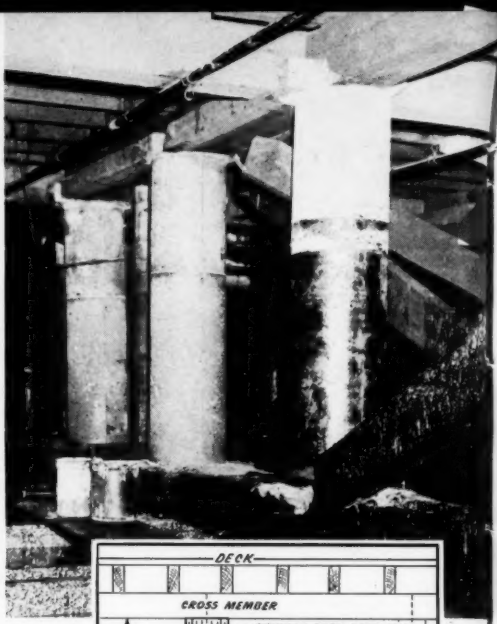
ACI News Letter

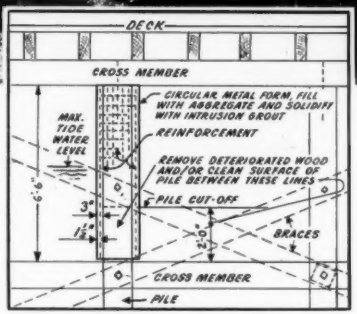
Vol. 23 No. 3 JOURNAL of the AMERICAN CONCRETE INSTITUTE November 1951





Typical pile showing rotted area above tidewater level.

Group of three piles shown after encasement with Prepakt Concrete. Note oil slick coating on pile in foreground.



Sectional drawing showing encasement procedure for repairing piles.

PREPAKT LOWERS COSTS OF TIDEWATER PIER REPAIR

Prepakt methods and materials were a natural answer to the problem of repairing rotted wooden piles supporting an eastern tidewater pier.

Substantial economies were effected with Prepakt methods by first removing all rotted materials and then encasing the tops of the piles with impermeable, high-strength Prepakt Concrete. Although the repair was carried out in an extremely "hard-to-get-at" position, the work was completed quickly and without interruption to pier traffic or removal of the pier decking for pulling old piles or driving new ones.

Future pile service is assured by the extreme durability of Prepakt Concrete in either salt or fresh water, plus its high resistance to freezing or thawing.

Full information on all Prepakt advantages as applied to your specific problems may be had by writing or calling the Main Office, Cleveland, Ohio.

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ACI

News Letter Contents

November 1951

■ On the Cover—Approximately 20,000 yards of Featherlite expanded shale aggregate will be used in the 40-story Republic Bank Building in Dallas. Architects, Harrison & Abramovitz, New York; associate architects, Gill & Harrell, Dallas; general contractor, J. W. Bateson Construction Co., Dallas.

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1951 ACI Awards go to Davis, Parsons, Price, McCoy, Caldwell and Safir

Raymond E. Davis, Douglas E. Parsons, Walter H. Price, W. G. McCoy, A. G. Caldwell and Otto Safir will receive ACI Awards from President Thomson at the 48th annual convention luncheon at the Netherland-Plaza Hotel, Cincinnati, Ohio, February 27, 1952. This announcement comes from the ACI Board of Direction following approval of the report of the Awards Committee.

The Turner Medal will go to Raymond E. Davis. The Henry C. Turner Medal was founded in 1927 by Henry C. Turner, ACI past president. It is awarded for notable achievements in or service to the concrete industry. It may be awarded once in any year but not necessarily in each year.

Douglas E. Parsons is the choice for the Alfred E. Lindau Award. This award was founded in 1947 by the Concrete Reinforcing Steel Institute in honor of the late Alfred E. Lindau, past president of ACI, and is in the form of a bronze plaque bearing a bas-relief portrait of Mr. Lindau. It is given only for outstanding contributions to reinforced concrete design practice and is not mandatory each year.

W. H. Price will receive the Wason Medal for "the most meritorious paper" of the year,

Factors Influencing Concrete Strength

which appeared in the February 1951 ACI JOURNAL.

W. J. McCoy and A. G. Caldwell will receive the Wason Medal for "noteworthy research" reported in

New Approach to Inhibiting Alkali-Aggregate Expansion

in the May 1951 ACI JOURNAL.

The Wason awards, established in 1917 by the

Continued on p. 4

late Leonard C. Wason, ACI past president, consist of a bronze medal and certificate to each awardee. For material published in the latest volume of *Proceedings* of the American Concrete Institute, an award is made for the "most meritorious paper." The research medal is given for original research work on a subject relevant to Institute activity and which, within the year, has been the subject of an Institute paper.

Otto Safir will receive the Construction Practice Award for his paper

Precast Concrete Construction in Canada

which was published in the February 1951 ACI JOURNAL.

This award was established in 1944 by the American Concrete Institute to recognize the man on the job for his resourcefulness in translating design into the completed structure.

The Award Winners

Raymond E. Davis—Turner Medalist

Raymond E. Davis, ACI member since 1926, has long been active in Institute affairs. Past president of the Institute, author or co-author of more than a dozen ACI papers, twice a Wason medalist (1931, 1934) and winner of the Construction Practice Award (1948), he is professor of civil engineering and director of the Engineering Materials Laboratory at the University of California.

After graduating from the University of Maine in 1911 with a BS degree, he began teaching at the University of Illinois where he received his MS degree in 1916. After a year at the University of Nebraska as associate professor of civil engineering, he joined the faculty of the University of California in 1920. In 1936, the University of Maine named him an honorary Doctor of Engineering.

Vice-chairman of ACI Committee 115, Research, and a member of ACI Committee 207, Properties of Mass Concrete, Professor Davis is affiliated with ASCE, ASTM, Sigma Xi, Tau Beta Pi, Chi Epsilon and Delta Signal Chi and is a fellow of the American Association for the Advancement of Science.

Douglas E. Parsons—Alfred E. Lindau Awardee

Douglas E. Parsons, chief of the Building Technology Division, National Bureau of Standards, Washington, D. C., is active in the affairs of many engineering associations. He is chairman of ASTM Committee C-15 on Masonry Units, ASA Committee A-41 on Masonry, and the Federal Specifications Technical Committee on Precast Reinforced Concrete Products.

Author of many papers in the Bureau's publications as well as in technical journals, in 1936 he received the ACI Wason Medal for noteworthy research reported in "Tests of Mesnager Hinges," published in the ACI JOURNAL.

An active ACI member since 1926, Mr. Parsons served on the Board of Direction, Advisory Committee, Publications Committee; was president in 1945, and chairman of the Technical Activities Committee in 1948. At present he is serving on ACI Committees 115 (Executive Group), 318, 323 and 326 (a joint ASCE-ACI committee).

Walter H. Price—Wason Medalist

Walter H. Price, head of the Materials Laboratory, U. S. Bureau of Reclamation, Denver, Colo., has been an ACI member since 1937 and is chairman of two ACI committees—210, Resistance to Erosion in Hydraulic Structures, and 613, Recommended Practice for Proportioning Concrete Mixes.

Since graduation from Tulane University in 1930, Mr. Price has been employed by the USBR. His work with the Bureau has included hydraulic model studies in connection with spillway designs of Boulder, Madden and other dams, and two years on the structural design of Boulder Dam spillways and power plant. Since 1943, his work has been devoted entirely to materials testing and research.

W. J. McCoy and A. G. Caldwell—Wason Medalists

W. J. McCoy, director of research, Lehigh Portland Cement Co., Coplay, Pa., graduated from Wisconsin State College and received a master's degree from the University of Minnesota. He has been with Lehigh Portland Cement Co. since 1938. A member of ACI Committee 115, Research, he is also active on ASTM committees on cement, concrete and mortars. Mr. McCoy has been affiliated with ACI since 1946.

A. G. Caldwell, research chemical engineer, Lehigh Portland Cement Co., is a graduate of the University of Illinois and has a master's degree from that university. Employed by Lehigh Portland Cement Co. since 1934, Mr. Caldwell devotes considerable time to studies of lightweight aggregates and volume change characteristics of mortar and concrete.

Otto Safir—Construction Practice Awardee

Otto Safir, after graduation in engineering in 1931, did post-graduate work in Switzerland on reinforced concrete bridges. Then, until 1945, he worked with consulting engineers and designing contractors in England and had his own consulting practice from 1945-1949. His main interests have been structural development and he has been responsible for the design of several reinforced concrete bridges. In 1945 he designed one of the first precast structures in England, a double-bay shed with two 75-ft spans. He was also associated with various precast concrete housing and factory projects in England. Since moving to Canada in 1949, he has engaged in similar work there, notably in British Columbia.

G. Nelson Perry

G. Nelson Perry, head of the consulting civil engineering firm of G. Nelson Perry and Co., Inc., Boston, Mass., died recently. He graduated in 1929 in civil engineering from Northeastern University and joined the J. R. Worcester and Co. consulting orgapization as a draftsman and was its senior designer when the firm broke up in 1946. He then became associated with the Thomas Worcester staff. In 1950, Mr. Perry formed his own firm of structural engineers in Boston. He had been an ACI member since 1949.

Andrew Weiss

Andrew Weiss, internationally known consulting engineer, died recently in Mexico City. He was born in Austria and was educated in the United States, receiving a degree of Engineer of Mines from the Colorado School of Mines in 1899. For many years, Mr. Weiss was engaged in reclamation work in Arizona, Nebraska and Wyoming.

In 1926, his engineering interests extended to Mexico where he served as resident engineer on the Don Martin Reclamation Project, which included building of the Don Martin storage dam and planning and construction of canals, distribution system and drainage of the entire 150,000-acre project.

From 1938 until his death, he was chief of the Technical Consulting Department, National Commission of Irrigation of Mexico, covering in its scope the problems of flood control, sanitary water supply, and the production of hydroelectric power.

In 1949, the Colorado School of Mines conferred upon him its medal for distinguished achievement in mineral engineering.

Mr. Weiss was a member of ACI, American Society of Agricultural Engineers, Association of Engineers and Architects of Mexico and an Honorary Member of ASCE.

James E. Sexton

James E. Sexton, architect, S. S. Kresge Co., Detroit, Mich., died recently. While staff architect for the S. S. Kresge Co. for the past 22 years, he designed numerous commercial buildings, notable among those of the last two years being store buildings in Syracuse, N. Y., Jamaica, Long Island, N. Y., Indianapolis, Ind., and an 8½-acre warehouse in Fort Wayne, Ind. Mr. Sexton had been an ACI member since 1930.

Howard R. Staley

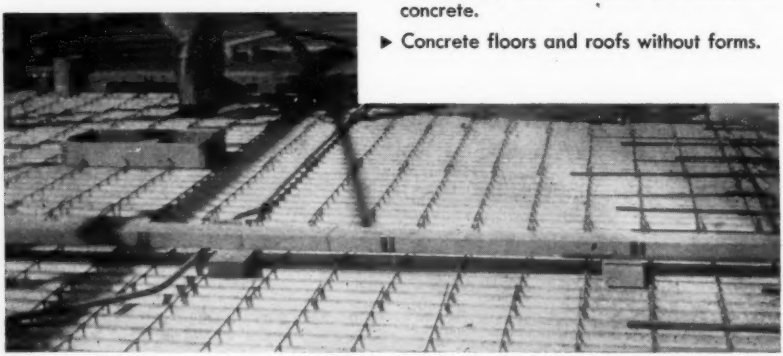
Howard R. Staley, construction engineer, Raw Materials Division, Atomic Energy Commission, Washington, D. C., died recently.

He attended Iowa State College for two years and received a BS in 1935 and an MS in 1937 from the Massachusetts Institute of Technology. A registered professional engineer in Maryland and Massachusetts, Mr. Staley was professor of building engineering and construction at MIT from 1935 to January 1951.

Throughout his entire professional life, he was an active member of ACI, ASTM and ASCE, as well as an outstanding consultant in the field of design and construction related to buildings. He had been an ACI member since 1937.

COFAR

- ▶ Reinforced concrete without forms.
- ▶ High-strength, deep-corrugated steel manufactured with welded, closely spaced transverse wires (T-wires).
- ▶ Positive reinforcement permanently anchored to and combined with structural concrete.
- ▶ Concrete floors and roofs without forms.



STRONG

Guaranteed yield point (.1% offset) over 80,000 psi effective as reinforcement, balanced design, up to full tensile strength over 100,000 psi, independent of form stress.

VERSATILE

Minimum 24 ga. —(A_s .35 sq. in. per foot width); maximum 18 ga. —(A_s .73 sq. in. per foot width); to meet full range concrete slab design. COFAR reinforcement satisfies all normal concrete slab design.

Cold-drawn, high-strength T-wires not over 6″ c.c. are welded to the deep-corrugated steel, in manufacture, and constitute temperature reinforcement in the slab, mechanical anchorage and positive shear transfer from concrete to steel.

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COFAR sheets hot-dip galvanized, with specification heavy galvanizing, provide complete permanence for normal interior exposures. Concrete and steel permanently bonded by chemical union of calcium zincate for all normal loads. Fire resistance for any exposure by lightweight, modern ceiling plaster or fibre protection.

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COFAR ***is concrete reinforcement,*** steel weight substantially equal to conventional reinforcement; COFAR ***also is the concrete form.*** Form sheeting as well as concrete runways are eliminated. COFAR comes detailed and cut to fit each job. Important material, labor, and time savings are made.

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Who's Who This Month

F. Thomas Collins and Earl M. Bennetsen

"Tilt-Up Construction Costs," p. 197, si the second recent JOURNAL paper dealing with tilt-up construction. (See also "Tilt-Up Construction in Western United States," October 1951, ACI JOURNAL, p. 133.)

F. Thomas Collins, consulting engineer, San Gabriel, Calif., is a recognized authority on precast concrete design and construction.

Earl M. Bennetsen, chief engineer, Wm. J. Moran Co., Alhambra, Calif., has been connected with building, construction and concrete work since he was 12 years old and now is regarded as one of the foremost authorities on tilt-up construction cost on the Pacific coast. A graduate of the University of Minnesota with a BS in aeronautical engineering, Mr. Bennetsen is a registered civil and mechanical engineer in California. He has been an ACI member since 1949.

E. W. Scripture, Jr., S. W. Benedict and F. J. Litwinowicz

"Effect of Temperature and Surface Area of the Cement on Air Entrainment," p. 205, is co-authored by E. W. Scripture, Jr., S. W. Benedict and F. J. Litwinowicz.

E. W. Scripture, Jr., vice-president in charge of research, The Master Builders Co., Cleveland, Ohio, has been an ACI member since 1931 and has contributed frequently to the JOURNAL as author and discusser of papers. He attended Harvard University where he obtained his BA degree in 1920, MA in 1922 and PhD in 1924. From 1923-1930, Dr. Scripture served as research chemist for a number of organizations and in 1930 became associated with The Master Builders Co. Besides his membership in ACI, he is affiliated with ASTM, American Chemical Society, Cleveland Engineering Society, American Ceramic Society, Society of Chemical Industry and Alpha Chi Sigma.

S. W. Benedict, director of research, The Master Builders Co., graduated from the University of Illinois in 1936. He served as junior materials engineer in the concrete research branch of the Illinois Division of Highways' Bureau of Materials until 1939 and as materials engineer in the Cement and Concreting Materials Section, National Bureau of Standards, until 1949 except for three years' service with the Army Air Force during World War II which included a year's assignment as structural engineer with the National Advisory Committee for Aeronautics. Mr. Benedict left the National Bureau of Standards in 1949 when he joined the staff of The Master Builders Co. He has been an ACI member since 1941.

F. J. Litwinowicz is senior research assistant in charge of the physical testing section

Continued on p. 8

WANTED

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JOURNAL OF THE AMERICAN CONCRETE INSTITUTE

Recent Civil Engineering Graduate preferred with experience on staff of college magazine or newspaper.

State salary expected and send appropriate personal, educational and experience data to

Secretary-Treasurer

American Concrete Institute 18263 W. McNichols Rd. Detroit 19, Michigan

of the Master Builders Co. research laboratory where he has been employed since 1946. Author of two other JOURNAL papers, he has been an ACI member since 1948.

Paul Rogers

Paul Rogers, structural engineer, Chicago, Ill., presents "Foundation for a Large Turbogenerator," p. 213.

Mr. Rogers has worked as structural engineer with Ralph H. Burke, Airport Consultant, Roberts and Schaefer Co., Hiram Walker and Sons, Inc., A. J. Boynton and Co., Public Utility Engineering and Service Corp., and Three Sisters, Inc. He also has served as consulting engineer on a number of projects in this country and in Transylvania and was assistant professor of structural engineering at the Chicago Technical College from 1940-45.

A graduate civil engineer from the Ecole du Genie Civil, Paris, France, Mr. Rogers also studied in the Graduate School, Illinois Institute of Technology. At present, he is a part-time lecturer on reinforced concrete at the Illinois Institute of Technology.

Mr. Rogers is a registered structural and professional engineer in Illinois, a registered civil engineer in California and is affiliated with ACI, ASCE, Western Society of Engineers, International Association for Bridge and Structural Engineering and the Societe des Ingenieurs Civils de France.

R. M. Mains

"Measurements of the Distribution of Tensile and Bond Stresses Along Reinforcing Bars," p. 225, introduces R. M. Mains, engineer and assistant group supervisor, Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Md.

Dr. Mains graduated from the University of Colorado in 1938 with a BS in civil engineering, received a MS degree from the University of Illinois in 1940 and a PhD from Lehigh University in 1946.

Since his graduation from college, he has been continually engaged in structural research and has taught courses in concrete design, structural analysis and foundation structures. Dr. Mains is especially interested in dynamic problems as they effect design.

L. H. Tuthill, R. E. Glover, C. H. Spencer and W. B. Bierce

"Insulation for Protection of New Concrete in Winter," p. 253, is the joint contribution of L. H. Tuthill, R. E. Glover, C. H. Spencer and W. B. Bierce.

L. H. Tuthill, research engineer, USBR, Denver, Colo., has been an active ACI member since 1926. He was awarded the ACI Construction Practice Award in 1945 for his paper, "Concrete Operations in the Concrete Ship Program." Since graduation from Oregon State College in 1920, Mr. Tuthill has been continuously engaged in design and construction work on irrigation, water supply projects and the building of dams, with the technique and control of concrete as his primary and absorbing interest. Since 1939 he has been a member of the staff of the Bureau of Reclamation.

R. E. Glover, research engineer, USBR, Denver, Colo., has been associated with the Bureau since 1924. His responsibilities are very largely with problems of design which require a knowledge of mathematics, such as supplying formulas and devising methods for analyzing the structural action of arch dams, conduction problems, etc. Mr. Glover has been an ACI member since 1936.

C. H. Spencer, construction engineer, USBR, Columbia Falls, Mont., began his engineering career with the U. S. Army Engineers in France during World War I. He has been employed by the Bureau of Reclamation since 1919 serving as assistant engineer on the Echo Unit of the Salt Lake Basin Project, Coalville, Utah; engineer, Gooding Division, Shoshone, Idaho, Project; resident engineer, Agency Valley Dam, Beulah, Ore.; construction engineer, Burnt River Project, Unity, Ore.; construction engineer, Fruitgrowers Project, Delta, Colo.: and resident and construction engineer, Deschutes Project, Bend, Ore. Since October 1947, Mr. Spencer has been in charge of all work involved in construction of the Hungry Horse Dam and power plant.

W. B. Bierce, assistant chief of construction, Division of Design and Construction, USBR, Amarillo, Tex., has been associated with the problem of protection of new concrete in winter for 10 years in private industry and for the past 16 years with the Bureau

Continued on p. 12

Another job better done with

Air-entrained concrete, made with Darex AEA, was used for this new Water Treatment Plant at Antioch, California.

Air Entraining Agent for all Concrete

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HAAS & ROTHSCHILD
Engineer:
JOHN S. BATES
Testing:
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On job after job these facts have been proved: Darex AEA concrete places easier and faster, finishes better, has better surface texture, is more homogeneous, thus more watertight.

At no extra cost, ready-mix concrete plants everywhere can furnish concrete that offers you these same advantages . . . concrete made with Darex AEA. Ask your supplier, or write to us for full details.

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The building with sloping floors in the foreground, and the fifteen story structure in the background are two of many buildings in the great new plant of the University of Mexico, Mexico City, built with Pozzolith Concrete. Contractors on this job say that Pozzolith, the Master Builders' product which disperses cement, reduces water and entrains the optimum amount of air, has enabled them to obtain better concrete at a substantial savings. In both dump truck transported concrete, and pumped concrete — used in the 140,000 seat stadium — Pozzolith made placing easier, avoided segregation and produced strengths higher than specified. Other advantages obtained with concrete produced with Pozzolith are — reduced shrinkage, high bond to steel, improved appearance and great durability.

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New Members

44 Individual, 1 Corporation, 5 Junior and 4 Student applications approved for October bring the Membership total to 5382. Top states were California—7, New York—6 and Pennsylvania—4. 15 new members were from outside the United States.

Individual

ASHLEY, SPRAGUE W., Cleveland, Ohio (Reinf. Conc. Design, Arthur McKee & Co.)
BAXTER, EDWARD D., Buffalo, N. Y. (Plant & Struct. Design, A. E. Baxter Engrg. Co.)
BEATTIE, IRA M., Fredericton, N. B., Canada (Prof., Univ. of New Brunswick)
BOLLEN, R. E., Lincoln, Nebr. (Testing Engr., Nebr. Dept. of Roads & Irrigation)
BROWN, WARREN E., Baton Rouge, La. (Chf. Chem., Ideal Cement Co.)
BROWNELL, WALTER KEITH, Los Angeles, Calif. (Engrg. Mgr., Gibbs & Hill, Inc.)
BURRIDGE, GORDON J., Braddon, Canberra, ACT, Australia
DAWE, HAROLD F., Denver, Colo. (Chf. Engr., Oil & Gas Div., Stearn Roger Mfg. Co.)
DIAZ CHAVEZ, Francisco, Caracas, Venezuela (Gen. Civil Engrg.)
DIEFENDORF, R. A., Bountiful, Utah (Supervision & Inspection, Bechtel Corp.)
DONOVAN, DENNIS E., Oklahoma City, Oklahoma (Arch. & Engr.)
DORSEY, HUGH D., Charlotte, N. C. (Struct. Engr., Easterby & Mumaw, Inc.)
EVANS, DON, Sacramento, Calif. (Constr. Engr., Calif. Div. of Hgwys.)
GAZON, HENRY J., New York, N. Y. (Engrg., Design & Development, Garco Design Corp.)
GORELICK, SAM, Washington, D. C. (Struct. Engr., Potomac River Naval Command)
GREEN, ALLEN I., Sewickley, Pa. (Secy. Treas., Green Engrg. Co.)
GUEST, WALTER C., Anderson, S. C. (Bldg. Constr., C. M. Guest & Sons)
GYI, KHIN M., Rangoon, Burma (Ass't. Project Engr., Port Authority of Rangoon)
HAWAYEK CURET, Jose, Hato Rey, P. R. (Sales & Service Engr., MacMurray & Co.)
HUNTER, HARRY G., Atlanta, Ga. (Struct. Engr.)
HVEEM, F. N., Sacramento, Calif. (Materials & Research Engr., Calif. Div. of Hgwys.)
INGRAM, A. A., New York, N. Y. (Chf. Engr., Marcus-Parkinson Constr. Corp.)
KASTEN, HENRY L., Edmonton, Alta., Canada (Cons. Struct. Engr.)
LAMB, GILES, Los Angeles, Calif. (District Materials Engr., Calif. Div. of Hgwys.)
LEITNER, HANS, Quito, Ecuador (Cons. Engr.)
LOZOVIC, SARAH, Tel-Aviv, Israel (C. E. & Quantity Surveying, Horman & Einziger)
MARUSHKO, WILLIAM N., Chicago, Ill. (Struct. Design in Concrete & Steel, S. F. Nydam)
MCBRIDE, WM. E., Jr., River Forest, Ill. (Struct. Engr., MacDonald Engrg. Co.)
MCCORMACK, DONALD E., Forest Hills, N. Y. (Draftsman & Field Super., American 3-Way Luxfer Prism Co., Inc.)
MITCHELL, RUSSELL S., Slippery Rock, Pa. (Civil & Mining Engrg.)
MOORE, JOSEPH HERBERT, State College, Pa. (Asst. Prof., Pennsylvania State College)
MYLNE, ROBERT S., Kentfield, Calif. (C. E., Dept. of the Air Force)
NEWELL, H. WINGRAVE, London SW, England (Technical Dir., Lenscrete Ltd.)
REECE, JAMES WILLIAM, Ottawa, Ont., Canada (Plant Supt., Harry Hayley & Sons)
SCHREIBER, GEORGE H., La Mesa, Calif. (Bldg. Insp., County San Diego)
SCOTT, ROBERT S., Alpena, Mich. (Cons. C. E., Scott Engrg. Co.)
SMITH, WM. F., Houston, Texas (Acting Chf. Chem., Ideal Cement Co.)

Continued on p. 12

Changes of Address

Please help us serve you better and trim expenses at the same time. It is of extreme importance that we receive prompt and accurate notice of a change of address. JOURNAL envelopes are addressed by machine the first week of each month and only by typing, an expensive and laborious process by comparison, can a last minute change be made to assure you of getting your JOURNAL.

Of more concern are the Members who wait until after moving to notify us, unwittingly costing ACI more than $100 per year in extra postage. The Post Office Department will not forward JOURNALS or Directories; they are returned to ACI C. O. D. and then additional postage must be affixed to mail them to the new address. A Directory sent to the wrong address in California costs ACI 54c—18c for each of the three trips; and with new postage rates in effect October 1, 1951 each trip will cost 27c.

Please help us get your publications to you at the regular time by notifying us four to six weeks in advance.—Editor.

SOBOROWSKI, Z. W., Montreal, Que., Canada (Reinf. Conc. Engr., Aluminum Co. of Canada)
TAYLOR, SAMUEL J., JR., Tampa, Fla. (Charge of Testing Lab., U. S. Engineers)
THOMPSON, ISADORE, Atherton, Calif. (Cons. Struct. Engr.)
TRIMBLE, CHARLES, Charleston, Ill. (Arch. & Constr. Supt., Ill. Div. of Arch. & Engrg.)
WILDE, ROBERT E., Chicago, Ill. (Assoc. Editor, *Rock Products*)
WILSON, JAMES B., Honolulu, T. H. (Arch. Engr., Law & Wilson)
WOLKOFF, ALEXANDER, New York, N. Y. (Struct. Designer, Robert W. Briggs)

Corporation

LOEDDING ENGINEERING CO., INC., Ambridge, Pa. (P. Herbert Loedding, Pres.)

Junior

BRANT, AUSTIN E., JR., Arverne, Queens, N. Y. (Drafting, Designing, Knappen, Tippetts, Abbett Engrg. Co.)
CAOLO, VITO, Springfield, Mass. (Struct. & Arch. Designing, Alderman & MacNeish)
ROBERTSON, EDGAR LEE, New Orleans, La. (Supplement Office Work with Inspection, E. B. Ludwig Construction Co.)
TAWEEL, FERDINAND JAMEEL, Springfield, Ill. (C. E., Ill. State Hwg. Dept.)
WYMAN, DONALD, Shawinigan Falls, Que., Canada (Civil & Field Engr., Shawinigan Engrg. Co.)

Student

CARLSSON, CARL F., Notre Dame, Ind.
DEUTSCHMAN, J. ALEJANDRO, Guatemala, Guatemala
LOPEZ, ANTONIO GRANA, Vedado, Havana, Cuba
MOODY, KEITH G., Urbana, Ill.

Honor Roll

February 1—September 30, 1951

Three newcomers—Jacob Adler, E. L. Howard and LeRoy A. Thorssen—add their names to the honor roll this month. Eddy Hernandez and J. E. Jellick close in a bit on the front-running Morgans. Put your name on this list of those who are introducing their friends and associates to the benefits of Institute membership.

Newlin D. Morgan (Ill.)	29
Newlin D. Morgan, Jr. (Mich.)	15½
Eddy Hernandez (La.)	10
J. E. Jellick (Calif.)	7½
L. G. Farrant (Fla.)	6
H. J. Gilkey (Ia.)	6
Samuel Hobbs (Calif.)	6
Oscar A. Nunez G. (Venezuela)	6
James A. McCarthy (Ind.)	5
Frank Kerekes (Ia.)	4
Alexander Klein (Calif.)	4
Jose A. Vila (Cuba)	4
Fernan Rodriguez-Gil (Venezuela)	3½
Rafael Ruiz (Guatemala)	3½
Oskar Schreier (N. Y.)	3½
George D. Youngclaus (Calif.)	3½
Oliver J. Julian (Mass.)	3
Alvin C. Loewer, Jr. (Pa.)	3
Ivan M. Viest (Ill.)	3
Jacob Adler (Israel)	2½
T. C. Kavanagh (Pa.)	2½
Donald Agrimson (S. D.)	2
J. F. Baxter (N. Y.)	2
Robert G. Deitrich (Md.)	2
Frank L. Ehasz (N. Y.)	2
E. L. Howard (Calif.)	2
Thomas J. Reading (S. D.)	2
C. P. Siess (Ill.)	2
Howard Simpson (Mass.)	2
Harry F. Thomson (Ill.)	2
LeRoy A. Thorssen (Canada)	2

Who's Who This Month

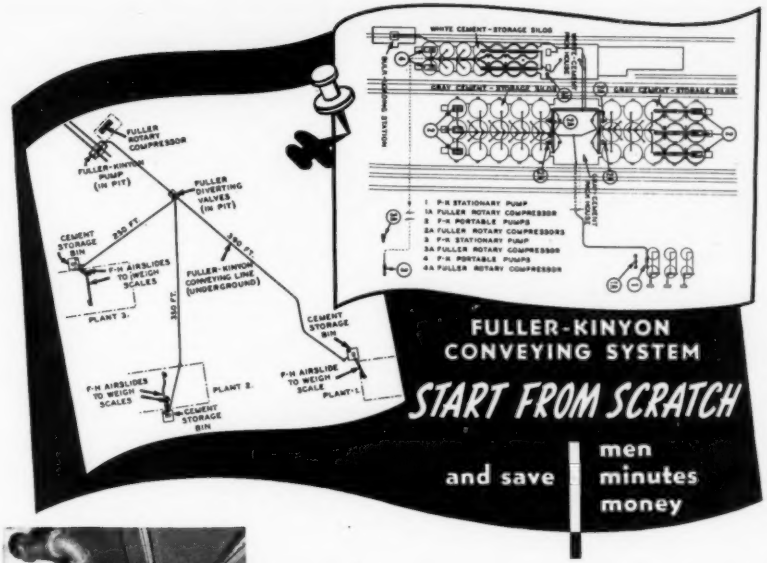
Continued from p. 8

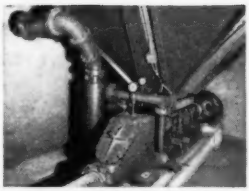
of Reclamation. During this period, the use of steam, salamanders, tarpaulins, Sisalkraft paper, earth material and fiber glass blankets for winter protection of new concrete have been used with varying degrees of success in protecting such structures as flood control and irrigation, bridges, siphons, concrete lining in canals, dams, office buildings, etc. Various field tests were carried out by the Bureau of Reclamation under Mr. Bierce's direction as construction engineer on the Fort Sumner Project, New Mexico, during the winter season of 1950-51, on the use of insulation, such as fiber glass mats, for protection of new concrete.

Construction conservation program

In the face of the critical shortage of structural and reinforcing steel and other controlled materials used in building construction, the Defense Production Administration has adopted a program devised by its Conservation Coordinating Committee to conserve scarce materials and thus allow for more construction in 1952 with the materials available.

To accomplish this, a set of standard design practices was recommended by the National Production Authority for use by architects and engineers in the construction industry. "Building Code Requirements for Reinforced Concrete, ACI 318-51" was recommended for use in designing reinforced concrete structures. Other recommendations included the use of wall-bearing masonry construction in one, two or even three-story buildings, flat slab construction with square or rectangular tied columns, use of noncritical materials to meet fireproofing requirements and use of lightweight concrete and plaster aggregates to reduce loads and thus the size of load-bearing members and foundations.





Fuller-Kinyon Stationary Pump conveying from bin

Fuller-Kinyon Unloader unloading from box car

The best way to solve any problem of conveying dry pulverized and granular materials is to "Start From Scratch". Get all the facts on pneumatic conveying systems while you are still in the planning stage, so that the conveying system can be designed to meet specific requirements of the job, to conform to local conditions, to insure minimum cost for erection, maintenance, and supervision. Fuller, as builders of four basic types of pneumatic conveyors, is able to provide the exact system best suited to individual needs.

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Fuller

P-133

Positions and Projects — ACI Members

Research

The assignment of Committee 115, Research, "is to review and correlate research in concrete and reinforced concrete and to consider research methods and objectives." The committee has attempted to fulfill this assignment through the Research Sessions at the annual conventions, through the distribution of the annual compilation of research projects underway at the various laboratories, and through limited correspondence among committee members.

From the continuing interest of the ACI membership in the activities of Committee 115, it is evident that real needs are being met. The work of any committee, however, is dependent not only on the activities of its members but also on the help and cooperation given it by all of the members of ACI. Consequently, we would greatly appreciate having your reactions and suggestions as to how we may better serve the interests of all; perhaps there are laboratories or projects which have been overlooked in sending out our recent requests for information; perhaps there are questions that should be raised or additional activities that should be started. Since the Committee is anxious to have complete coverage of all research projects in its next compilation, all persons having suggestions or knowing of omissions are urged to initiate correspondence at once. No publicity will be given any information without the permission of the correspondent. Address your suggestions and inquiries to:

Prof. George W. Washa
Secretary ACI Committee 115
Engineering Building
University of Wisconsin
Madison 6, Wisconsin

Hansen appointed research engineer

Howard Hansen, formerly head of the structures research department, Naval Civil Engineering Research Laboratory, Port Hueneme, Calif., has been appointed research engineer in the civil engineering department, Illinois Institute of Technology, Chicago.

Hansen received a BS in civil engineering from Purdue University in 1930 and a MS from Tulane University in 1939. He has taught at Tulane, Texas A & M and the University of Florida. He is a member of ACI Committee 323, Prestressed Reinforced Concrete, lumber standards committee, U. S. Department of Commerce and the advisory board, engineering mechanics division of ASCE.

Thompson & Lichtner name engineer

William F. M. Longwell has joined the Thompson and Lichtner Co., Inc., Engineers, Brookline, Mass., as a principal engineer. Educated at West Point, State University of Iowa and Yale University, he served more than 18 years with the U. S. Army Corps of Engineers and taught ROTC and civil engineering for 15 years. Major Longwell was formerly professor of civil engineering at Worcester Polytechnic Institute.

Pickett at Wisconsin

Gerald Pickett became professor of mechanics at the University of Wisconsin with the beginning of fall term this year. Previously he was professor of applied mechanics at Kansas State College, Manhattan, Kansas.

Rogers returns from India

Franklyn C. Rogers, on leave for the past year from the post of professor of civil engineering at Rutgers University, has returned from India where he represented the Harza Engineering Co., Chicago, as resident engineer on the Maithon project of the Damodar Valley Corp. He also was director of engineering for the Damodar Valley Corp. Professor Rogers has resumed his duties at Rutgers University as director of the Joint Highway Research Project.

Blaw-Knox photo

Concrete paving equipment train in action on western extension of Pennsylvania Turnpike near Harmarville. Dual-drum paver in foreground is followed by spreader, finishing machines and (at extreme left) hand finishing and curing operations. This equipment, augmented by a bulk cement plant, aggregate batching plant and a vibratory subgrader not shown, enabled the contractor, Patterson Construction Co., to pave 200 ft per hour when conditions were good.

Blanks accepts new post

Robert F. Blanks, formerly chief, Research and Geology Division, U. S. Bureau of Reclamation, Denver, Colo., has accepted the position of vice-president and general manager of Great Western Aggregates, Inc., a subsidiary of the Ideal Cement Co., Denver.

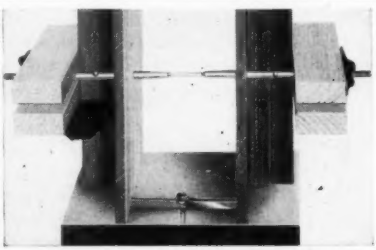
Great Western Aggregates, Inc., was formed in 1950 by Ideal Cement Co. with the objective, through research, of studying aggregates of all kinds on the theory that since cement must always be used with some type of aggregate to produce concrete, the ultimate future of the cement business rests largely on a complete and thorough knowledge and understanding of aggregates of all types.

Mr. Blanks has a long record of experience and accomplishments in the fields of civil engineering, research and concrete design. A past president of ACI and chairman, Committee 207, Properties of Mass Concrete, he has been associated with ACI since 1932.

Greulich receives certificate of appreciation

Gerald G. Greulich, consulting engineer, Pittsburgh, Pa., was presented with an Army emblem and Certificate of Appreciation "for outstanding patriotic service to the Department of the Army during World War II." Mr. Greulich was cited, in particular, for his wartime service in the Office of the Chief of Engineers in designing and making available

Continued on p. 16



Williams Clamps showing nail in stud spacing—waler support—and form aligner

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Continued from p. 15

the pierced steel landing mat "that proved to be an important factor in the successful conclusion of the war." The presentations were made by Brig. Gen. Gordon E. Textor, assistant chief of engineers for military operations.

ACI visitor

Norman B. Appleton, Managing Director, Appleton Industries, Ltd., Brisbane, Australia, visited briefly at ACI headquarters on October 5. Mr. Appleton, whose firm manufactures steel window sash and related products, was returning from England through Canada and the United States, studying the use of concrete for housing which his firm proposes to construct for employees migrating from England.

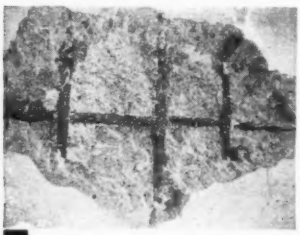
Mahler award at Stevens Institute

Tests on prestressed concrete will be continued this winter at Stevens Institute of Technology, Hoboken, N. J., by students working for the Mahler award, established last year to stimulate interest in the testing of materials. The award is offered annually by Martin Mahler, consulting engineer, Union, N. J.

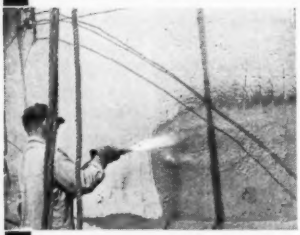
Mahler, a member of ACI, ASTM and other technical societies, is active in a movement toward an international standard notation, code and specification for prestressed concrete work.

Walter E. Hanson

Walter E. Hanson is now bridge engineer for the Illinois State Highway Dept., with headquarters at Springfield. Formerly he was associate professor of civil engineering at the University of Illinois.



Typical example of spalling. Note corrosion of reinforcing rods exposed by disintegrating concrete.



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5. The average number of copies of each issue of this publication sold or distributed, through the mails or otherwise, to paid subscribers during the 12 months preceding the date shown above was: (This information is required from daily, weekly, semiweekly, and triweekly newspapers only.)

HARVEY WHIPPLE
(Signature of editor)

Sworn to and subscribed before me this 24th day of September, 1951.

ETHEL B. WILSON, Notary Public
[SEAL] (My commission expires Dec. 19, 1954)

Texas uses shale aggregate

Quick acceptance of expanded shale aggregate in Texas is emphasized by its use in the construction of more than a dozen multistory buildings in the Fort Worth-Dallas area in the 2½ years since the publication of "Lightweight-Aggregate Concrete," in the May 1949 ACI JOURNAL. Prior to that time there was no structural concrete in that area in which expanded shale had been used as the aggregate.

Approximately 20,000 cu yd of expanded shale aggregate, produced by the Featherlite Corp., Dallas, will be used in the 40-story Republic Bank Bldg. (see News Letter cover) which is scheduled for completion in 1953. The expanded shale aggregate will be used for structural concrete, fireproofing of supporting members and for fill over structural concrete.

Corps of engineers laboratory installs x-ray diffraction apparatus

The Waterways Experiment Station, Corps of Engineers, U. S. Army, has recently added x-ray diffraction apparatus to the facilities at its Concrete Research Division, Jackson, Miss.

The principal studies in which the apparatus will be initially employed are investigations for the Office, Chief of Engineers designed to develop data that will increase quality and reduce cost of concrete used in the Civil Works program.

Studies by other agencies including the Bureau of Reclamation laboratories, Denver, Colo.; the Portland Cement Assn. Fellowship at the National Bureau of Standards, Washington, D. C. and the Kentucky Highway Department Research Laboratories, Lexington, Ky., have indicated that x-ray diffraction methods can be a powerful tool in identifying, classifying, and understanding the properties of materials used in concrete construction.

The apparatus, the latest model XRD-3 developed by the General Electric X-Ray Co., Milwaukee, Wis., is designed to be used either for making photographic records of x-ray diffraction patterns or for the direct recording of diffracted rays on a strip chart.

One valuable characteristic of the apparatus is that very small samples weighing only a few hundred-thousandths of an ounce are used to obtain the diffraction data. This permits selection of tiny portions of very fine grained materials such as admixtures for concrete, cements, hydrated cements, and clays for study. Some of these materials cannot be satisfactorily studied and identified by microscopic or chemical methods.

Samson accepts new position

Charles H. Samson, Jr., has accepted a position as structures engineer, Consolidated Vultee Aircraft Corp., Fort Worth, Tex. He was previously on the teaching staff at the University of Missouri.

ALPHABETICAL LIST OF ADVERTISERS

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International Congress accepts resolutions

At the International Congress of Prestressed Concrete, September 10-13, 1951, Ghent, Belgium, several resolutions were accepted.

Being important to know how concrete acts and to have all information concerning its principal behavior, the Congress desired that in tests on concrete, special attention be given to study of elastic, plastic and viscous deformation of concrete; effect of repeated and alternating loadings; effect of the variations of temperature and moisture; and deformation of concrete under shear.

The Congress suggested that the laboratories undertake tests on the cracking of slabs prestressed in two directions and asked that Eugene Freyssinet coordinate the programs and collect the results with the idea of presenting an over-all picture.

To avoid delaying progress in prestressing, it was recommended that the tentative regulations on prestressing be sufficiently elastic to allow engineers, who apply the most recent scientific knowledge, to obtain waivers to these specifications, whenever they are justified. Finally, the Congress wanted a system of international notations worked out to simplify reading of technical papers.

Vacuum concrete pipes

Ferrocimento Co., Rome, Italy, is presently using the vacuum process to cast ten miles of concrete pipe nearly 10 ft in diameter to operate under a pressure of 180 psi. Cast in 16-ft lengths with 5-in. walls no steel cores are used and approximately one half of the pipes are prestressed. A 1:5 cement-aggregate mix is used which has resulted in average cylinder strengths of 7000 psi.

Other pipes manufactured by the company have 12-in walls from which the inside and outside forms are removed half an hour after casting. The pipe sections, weighing up to 50 tons, are lifted from the base rings by a vacuum lifter the morning after they are cast.

Engle appointed assistant vice-president

A. D. Engle, a member of the Austin Company's engineering staff since 1920 and its district engineer at Chicago since 1945, has been appointed assistant vice-president for research. Mr. Engle will be responsible for special engineering research work and certain foreign assignments, with headquarters at the company's general offices in Cleveland.

Title No. 48-19

Lateral Force Distribution in a Concrete Building Story*

By T. Y. LIN†

SYNOPSIS

A rational method for the distribution of lateral forces among vertical resisting elements of a concrete building story is presented. The basic theory is given together with examples, including the general case of walls and odd-shaped columns at skew angles to one another.

INTRODUCTION

Along the West Coast, and especially in California, there have been developed recently certain methods of analysis and design of buildings against earthquake forces. Because of the growing importance of lateral force design, it is deemed desirable to study and present some of the problems encountered. This paper describes a convenient rational method (hereinafter named the *rigidity center method*) for the distribution of lateral forces among different vertical resisting elements, such as columns and walls, in a concrete building story. The method involves the determination of the center of rigidity of the group of resisting elements, the location of principal axes of the group, and the computation of skew rigidities of the elements. While its application to simpler cases, *e.g.*, with round or square columns or with walls at right angles, is well known and established, its general solution, including odd-shaped elements at skew angles, is believed to be new. Analysis by this method reveals many interesting and important points which are often incorrectly ignored in design. Only the analytical problems are discussed here, however; problems concerning design are not considered.

All classical methods of lateral force analysis, such as the portal, cantilever, and moment distribution methods, deal only with individual vertical bents, assuming that each bent acts independently, unaffected by the rigidities and deformations of adjoining ones in the story. While such an assumption may be approximately true in a regular steel building with timber floors, they are certainly incorrect for complicated framing, especially where concrete floor slabs serve as rigid horizontal diaphragms. Major building codes along the West Coast now specify that the total lateral force at any horizontal plane shall be distributed to the various resisting elements in proportion to their

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†Member American Concrete Institute, Associate Professor of Civil Engineering, University of California, Berkeley, Calif.

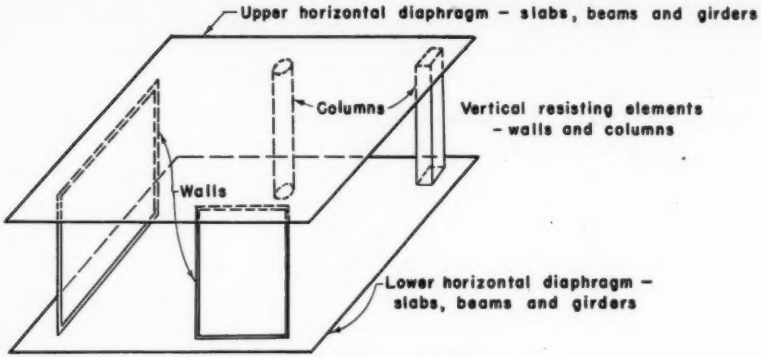

Upper horizontal diaphragm – slabs, beams and girders
Columns
Vertical resisting elements – walls and columns
Walls
Lower horizontal diaphragm – slabs, beams and girders

Fig. 1—Structural skeleton of a building story

rigidities and proper provision shall be made for horizontal torsional moments when the center of rigidity of resisting elements is not coincident with the center of application of lateral forces. Such requirements necessitate the development of a rational method for lateral force distribution among elements of a building story as described here.

DEFINITIONS AND SYMBOLS

Typical structural skeleton of a story of a building is shown in Fig. 1. At the top of the story is an upper horizontal diaphragm (consisting of slabs, beams and girders in the floor) transmitting lateral forces from the stories above to the walls and columns in the story which form the vertical resisting elements. The lateral forces are then transmitted to the lower horizontal diaphragms, which in general are also made up of slabs, beams, and girders.

Symbols used, most of which are shown in Fig. 2, are defined below.

1–1, 2–2 = major and minor axis, respectively, of any vertical resisting element, passing through the centroid of the section of the element.

X–X, Y–Y = principal axes of the group of vertical resisting elements, passing through the center of rotation of the group.

X'–X', Y'–Y' = any arbitrary pair of axes through any origin, chosen for convenience in computation.

ϕ = angle measured from X–X to 1–1

Δ = angle measured from X'–X' to X–X

Θ = angle measured from X'–X' to 1–1 = $\phi + \Delta$

R = rigidity of any element = the horizontal force to be applied to the element to produce a unit horizontal displacement of the top with respect to the bottom of the element in the direction of the applied force; R_1, R_2, R_x, R_y, R_x', R_y' = rigidities along the axes 1–1, 2–2, X–X, Y–Y, X'–X', Y'–Y', respectively.

Note: For a round or square column, all these R's are equal. R_x, R_y, R_x', R_y' being rigidities along axes other than the principal axes are termed skew rigidities.

R_{xy} = product of rigidity = accompanying horizontal force for unit displacement in a perpendicular direction.

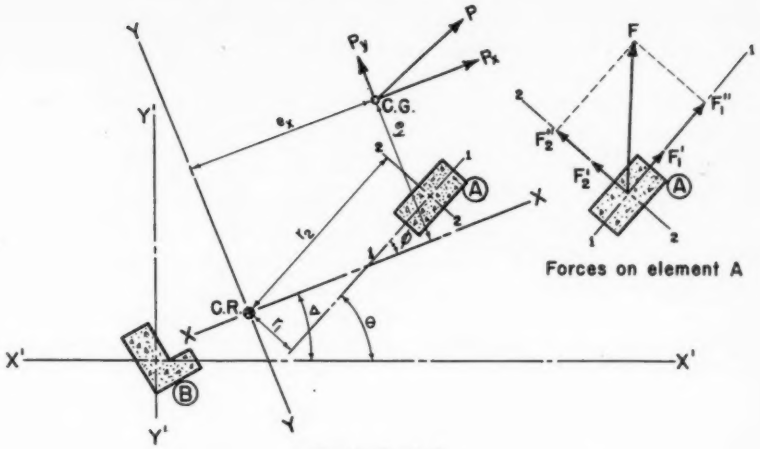


Fig. 2—Symbols

CR = center of rigidity = center of rotation due to pure moment acting on the horizontal diaphragm.

CG = center of mass, or center of application of force.

r_1, r_2 = perpendicular distances from CR to 1–1 and 2–2, respectively.

J = polar moment of rigidity of the group about $CR = \Sigma R_1 r_1^2 + \Sigma R_2 r_2^2$

F' = force on any element due to direct lateral load acting on the diaphragm through CR of the group; F_1', F_2' = its components along 1–1 and 2–2, respectively.

F'' = force on any element due to pure moment acting on the horizontal diaphragm; F_1'', F_2'' = its components along 1–1 and 2–2, respectively.

F = resultant force on any element due to the combined effect of direct and moment load; F_1 and F_2 = its components along 1–1 and 2–2, respectively.

P = resultant load acting on the diaphragm; P_x and P_y = its components along X–X and Y–Y, respectively.

e_x, e_y = eccentricities of P_y and P_x, respectively; measured from CR

M = moment acting on the horizontal diaphragm = $P_x e_y - P_y e_x$

Δ_x, Δ_y = translatory displacements between the upper and lower horizontal diaphragms along X–X and Y–Y due to P_x and P_y, respectively.

Q_x', Q_y' = resultant force to produce unit translation of the entire group along X'–X' and Y'–Y', respectively.

Signs for ordinates and angles follow the usual trigonometric notation. Clockwise moments, forces upward or to the right are considered positive. Arrowheads are used to denote the direction of acting forces.

ASSUMPTIONS

This analysis is based on the assumption that the upper and lower horizontal diaphragms are rigid in the horizontal plane, *i.e.*, the horizontal distortions of the diaphragms are negligible. This assumption will introduce only minor

errors if the floor slabs acting as beams in a horizontal plane are much more rigid than the vertical resisting elements in vertical planes, as is usually the case.

Relative rigidities of the different vertical elements can be obtained experimentally or analytically. It is beyond the scope of this paper to discuss their computation. Conventional formulas are available, some of which are listed in Appendix 1 for reference.

Computation of the force P acting on the story, including forces transmitted from upper stories, is not discussed here. It is assumed that the force is completely known and is entirely applied at the level of the upper diaphragm and resisted by an equal and opposite force in the lower diaphragm. For a force of variable direction, its magnitude and point of application being fixed, one example is solved in Appendix 2 illustrating the method for determining maximum load on an element.

GENERAL PROCEDURE

A general procedure for the distribution of lateral forces among vertical resisting elements of a story is outlined below. The derivation of the method together with the formulas used is explained later.

Step 1—Determine the relative rigidities of the different elements, each one in two directions along the principal axes of the section of the element. Round and square columns, having equal rigidities in all directions, need only one computation. Walls are usually assumed to be very flexible along the widths and hence only their rigidities along the lengths need be computed.

Step 2—Locate the center of rotation of the group of vertical resisting elements in the story.

Step 3—Locate the principal axes X–X and Y–Y of the group. For a group of round or square columns, any axis passing through the center of rotation is a principal axis. For rectangular columns or walls at right angles, the principal axes of the group are parallel to the principal axes of the columns or walls. For odd-shaped columns, or for rectangular columns and walls at skew angles, these must be located by computation, as shown later.

Step 4—Compute J of the group, $J = \Sigma Rr^2$ or $= \Sigma R_1{}^2r_1{}^2 + \Sigma R_2r_2{}^2$ ..... (1)

Step 5—For moment M, compute the force on any column as follows:

$$F_1'' = \frac{Mr_1}{J} R_1 \quad \text{..... (2)}$$

$$F_2'' = \frac{Mr_2}{J} R_2 \quad \text{..... (3)}$$

Step 6—Compute skew rigidities, R_x and R_y, hence ΣR_x and ΣR_y. Resolve direct load P into P_x and P_y. Then

$$\Delta_x = \frac{P_x}{\Sigma R_x} \quad \text{..... (4)}$$

$$\Delta_y = \frac{P_y}{\Sigma R_y} \quad \text{..... (5)}$$

For each resisting element, compute forces F_1' and F_2' by the following:

$$F_1' = R_1 (\Delta_x \cos \phi + \Delta_y \sin \phi) \quad \text{..... (6)}$$

$$F_2' = R_2 (\Delta_y \cos \phi - \Delta_x \sin \phi) \quad \text{..... (7)}$$

Step 7—Combine the components due to direct loads and moment vectorially:

$$F_1 = F_1' + F_1'' \quad (8)$$
$$F_2 = F_2' + F_2'' \quad (9)$$
$$F = \sqrt{F_1^2 + F_2^2} \quad (10)$$

CASE 1—ROUND OR SQUARE COLUMNS

Instead of solving the general case first, it is thought easier for the reader to start with a simple case of round or square columns having equal rigidities in all directions and hence any axis through the center of the column or the centroid of the group is a principal axis. Such a problem can be made analogous to the conventional solution for a riveted connection under eccentric load where the connecting plates are considered to be rigid and the rivets elastic. The stiffness of each column is analogous to the area of each rivet. Since the analysis for riveted connection under eccentric load is presented in any standard treatise on strength of materials or structural design, its derivation will not be repeated here.

Example 1

A story of a building has three round columns with I of section and L of member as given (Fig. 3). Columns A and C are fixed at top and bottom; column B is hinged at top and fixed at bottom. Lateral force transmitted through the story is given as 30 kips acting as shown. Determine the lateral force acting on column A.

Step 1—Referring to Appendix 1, the relative rigidities of the columns are computed as follows:

Column A: $R = 12\,EI/L^3 = 12\,E\,8000/20^3 = 12\,E$, Relative $R = 1$
Column B: $R = 3\,EI/L^3 = 3\,E\,27{,}000/15^3 = 24\,E$, Relative $R = 2$
Column C: $R = 12\,EI/L^3 = 12\,E\,3000/10^3 = 36\,E$, Relative $R = 3$
$\Sigma R = 6$

Step 2—The center of rotation for the group of columns is located at the centroid of the rigidities. Using a pair of reference axes, X'–X' and Y'–Y', we obtain

$x_0' = (2+3)24/6 = 20$ ft
$y_0' = 3\times30/6 = 15$ ft

which locate the center of rotation O.

Step 3—Since the group consists of round columns only, any rectangular axes through O can be taken as the principal axes. For convenience, choose X–X and Y–Y as shown.

Step 4—Compute J of the group as in Table 1.

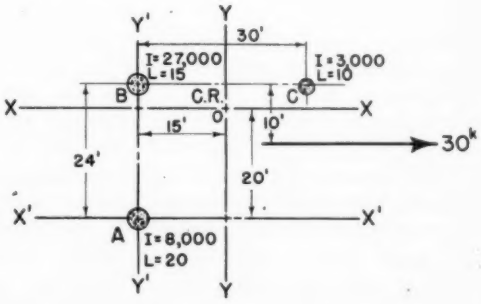


Fig. 3—Lateral forces acting on section of building story

Fig. 3a (below)—Resultant force acting on column A in Fig. 3

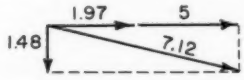


TABLE 1—COMPUTATION OF POLAR MOMENT OF RIGIDITY

Column	R	x	y	Rx^2	Ry^2
A	1	−15	−20	225	400
B	2	−15	+ 4	450	32
C	3	+15	+ 4	675	48
			Σ =	1350	480

$J = \Sigma R(x^2 + y^2) = 1350 + 480 = 1830$

Step 5—Moment $= Pe = 30 \times 6 = 180$ kip-ft. For column A, $R = 1$ in all directions, and any two perpendicular directions can be used as the pair of principal axes. Again, for convenience use X'–X' and Y'–Y' as the principal axes 1–1 and 2–2 of column A. Then

$$F_1'' = \frac{Mr_1}{J} R = \frac{180 \times 20}{1830} 1 = 1.97 \text{ kips}$$

$$F_2'' = \frac{Mr_2}{J} R = \frac{180 \times 15}{1830} 1 = 1.48 \text{ kips}$$

Step 6—$P_x = 30$ kips; $P_y = 0$. $\Sigma R_x = \Sigma R_y = 6$.

$\Delta_x = 30/6 = 5$; $\Delta_y = 0$; $\phi = 0$

$F_1' = R_1 (\Delta_x \cos \phi + \Delta_y \sin \phi) = 1(5 \times 1 + 0) = 5$

$F_2' = R_2 (\Delta_y \cos \phi - \Delta_x \sin \phi) = 1(0 \times 1 - 5 \times 0) = 0$

Step 7—Combine the effects of moment and direct load as in Fig. 3a. The resultant force acting on column A is therefore a lateral load of 7.12 kips acting in the direction shown.

CASE 2—RECTANGULAR COLUMNS AND WALLS AT RIGHT ANGLES

Rectangular columns are more complicated than round or square ones because they have different rigidities in all directions and hence are not analogous to rivets in a connection. It is well-known in strength of materials that when a load is applied to a rectangular beam oblique to its principal axes, the deflection of the beam will not be in the direction of the load. In other words, the oblique load must be resolved into components along the two principal axes, and the deflections along the two axes obtained separately and then combined vectorially. However, if all rectangular columns are arranged at right angles to one another, the principal axes of the group can be easily located, being parallel to the principal axes of the columns themselves. The center of rotation of the group can also be easily determined, being at the intersection of two perpendicular lines, each representing the resultant of the rigidities along one direction. Resultant force acting on the horizontal diaphragm must be resolved into a moment and two components along these two principal axes and distributed accordingly. This is illustrated in Example 2.

Walls differ from columns in that usually only the rigidities along their lengths are considered, while those along their widths are neglected. The rigidities of walls widthwise are so much smaller than those lengthwise that they need not be considered unless they serve as the main resisting elements in that direction which is seldom the case. If walls are at right angles to one another, they can simply be treated as the columns in Example 2, only simplified by the fact that each wall has a zero rigidity about one principal axis. It must be noted that since walls resist lateral forces only in one direction, their solution is not analogous to that of a welded connection where fillets are always assumed to resist forces in all directions.

Example 2

Three rectangular columns in a story are at right angles to one another

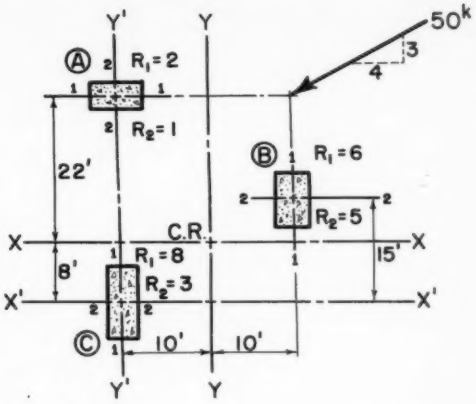
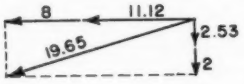


Fig. 4—Lateral forces acting on three rectangular columns at right angles to one another

Fig. 4a (below)—Resultant force acting on column A in Fig. 4



and carry a total lateral force of 50 kips (Fig. 4). The relative rigidities of the columns are given, as shown. Compute the load carried by column A.

Step 1—Rigidities having been given, they can be summed up as follows:

$$\Sigma R_x = 2 + 5 + 3 = 10$$
$$\Sigma R_y = 1 + 6 + 8 = 15$$

Step 2—Center of rotation located by

$$x_0' = 5 \times 20/10 = 10 \text{ ft}$$
$$y_0' = (1 \times 30 + 6 \times 15)/15 = 8 \text{ ft}$$

Step 3—Principal axes X–X and Y–Y are located through the center of rotation as shown.

Step 4—$J = \Sigma(R_1r_1^2 + R_2r_2^2) = (5 + 2 + 3)10^2 + 6\times7^2 + 1\times22^2 + 8\times8^2 = 2290$

Step 5—Resolving 50 kips into $P_x = 40$ kips and $P_y = 30$ kips, we obtain $M = -40 \times 22 + 30 \times 10 = -580$ kip-ft.

The forces acting on column A are:

$$F_1'' = \frac{Mr_1}{J}R_1 = \frac{580 \times 22}{2290}2 = 11.12 \text{ kips} \leftarrow$$

$$F_2'' = \frac{Mr_2}{J}R_2 = \frac{580 \times 10}{2290}1 = 2.53 \text{ kips} \downarrow$$

Step 6—$P_x = 40$ kips $P_y = 30$ kips

$$F_1' = (40/10)2 = 2 \text{ kips} \leftarrow$$
$$F_2' = (30/15)1 = 2 \text{ kips} \downarrow$$

Step 7—Combine the forces vectorially (Fig. 4a). $F = \sqrt{19.12^2 + 4.53^2} = 19.65$

We are likely only interested in the components along the principal axes, which are 19.12 and 4.53 as shown.

CASE 3—ODD-SHAPED COLUMNS AND WALLS AT SKEW ANGLES

Now consider the general case where vertical resisting elements of any shape are arranged at skew angles to one another. Basically, it requires a solution of three simultaneous equations to determine the three components of movement (two translatory and one rotational) between the upper and lower horizontal diaphragms. However, such a mathematical approach

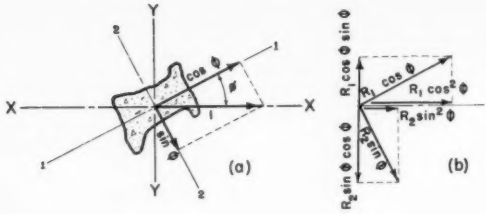


Fig. 5—Forces affecting skew rigidity of column

would not only be cumbersome but would also tend to conceal the design characteristics which can be so easily visualized by the rigidity center method. It might be interesting to note that while there is some apparent similarity between this method and the well-known elastic center method for arch analysis, the two are actually different problems, and this analysis is further complicated by the vectorial nature of the resisting elements.

Fundamental principles underlying the method are discussed in four parts as follows:

(1) The skew rigidity of a column or wall, *i.e.*, its rigidity in a direction not along a principal axis.
(2) Location of the center of rotation of the group with elements at skew.
(3) Location of the principal axes of the group with elements at skew.
(4) Distribution of forces along principal axes of each element.

Skew rigidity

Consider any column with any cross section (Fig. 5a). The principal axes of the element have been located and the rigidities along the principal axes are given as R_1 and R_2. Required: the rigidity of the element along any other axis X-X, making angle ϕ with 1-1.

Assume a unit translation of the column in the given direction X-X; this unit translation can be resolved into a translation of $\cos \phi$ along 1-1 and of $\sin \phi$ along 2-2. Hence the forces required to produce those two components of translation will be $R_1 \cos \phi$ and $R_2 \sin \phi$ along the principal axes. These two forces can be resolved into components acting parallel and perpendicular to X-X as shown in Fig. 5b. The resultant force along X-X will be the rigidity of the element in this direction (by definition, rigidity is the force in a given direction required to produce unit translation in that direction), hence

$$R_x = R_1 \cos^2\phi + R_2 \sin^2\phi \quad (11)$$

It can be seen that simultaneously a resultant force perpendicular to X-X must be exerted, and the total amount is given as

$$R_{xy} = R_1 \cos \phi \sin \phi - R_2 \sin \phi \cos \phi = (R_1 - R_2) \sin \phi \cos \phi \quad (12)$$

where R_{xy} might be called the product of rigidity. It is evident that R_{xy} is zero for round and square columns about any axis, and is zero for all columns about their own principal axes.

Similarly for a unit translation in the direction Y-Y, it can be shown that

$$R_y = R_1 \sin^2 \phi + R_2 \cos^2 \phi \quad (13)$$
$$R_{xy} = (R_1 - R_2) \sin \phi \cos \phi \quad (12)$$

In the case of walls, R_2 is assumed to be zero, and the formulas reduce to

$$R_x = R_1 \cos^2 \phi \quad (14)$$

$$R_y = R_1 \sin^2 \phi \quad (15)$$

$$R_{xy} = R_1 \cos \phi \sin \phi \quad (16)$$

Location of the center of rotation

To determine the center of rotation of a group of elements at skew angles proceed as follows.

Choose any convenient pair of axes, X'-X' and Y'-Y'. Assume a unit translation of the entire group along the direction X'-X'. Then for each column, there will be two components of force acting on it, *i.e.*, R_x', and R_{xy}', the values of which are computed from Eq. 11 and 12. Locate the resultant of these components of all the elements—this can be done either graphically or algebraically. That resultant is given as Q_x' in Fig. 6.

Next assume a unit translation of the entire group along the direction Y'-Y'. Compute the components of force acting on each column. Locate the resultant of all these components. That resultant is given as Q_y', Fig. 6.

The intersection of the resultants Q_x' and Q_y' is the center of rotation of the entire group because any force P applied through this point of intersection can be resolved into components along directions of Q_x' and Q_y', producing pure translations along X'-X' and Y'-Y', but no rotation. It can be seen easily that for a translation along any third direction, the resultant force also will pass through the CR as determined by the intersection.

Location of principal axes

After locating the center of rotation, the next step is to locate the principal axes of the group. As can be seen from the above, a force applied through the center of rotation may not produce a translation only in that direction, unless the force is applied along either of the principal axes of the group, about which $\Sigma R_{xy} = 0$. Referring to Fig. 2 and 5, we have

$$\Sigma R_{xy} = \Sigma (R_1 - R_2) \sin \phi \cos \phi = 0.$$

Since $\phi = \Theta - \Delta$, we have

$$\Sigma (R_1 - R_2) \sin (\Theta - \Delta) \cos (\Theta - \Delta) = 0$$

$$\therefore \quad \Sigma \frac{R_1 - R_2}{2} \sin 2(\Theta - \Delta) = 0$$

$$\therefore \quad \Sigma \frac{R_1 - R_2}{2} \left[\sin 2\Theta \cos 2\Delta - \cos 2\Theta \sin 2\Delta) \right] = 0$$

Δ being a constant, we have

$$\cos 2\Delta \, \Sigma \frac{R_1 - R_2}{2} \sin 2\Theta - \sin 2\Delta \, \Sigma \frac{R_1 - R_2}{2} \cos 2\Theta = 0$$

Hence,

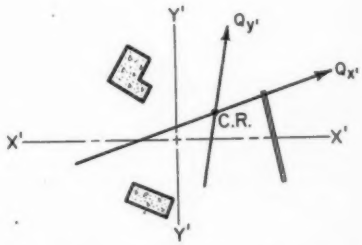


Fig. 6—Determination of center of rotation of a group of elements at skew angles

$$\tan 2\Delta = \frac{\Sigma \frac{R_1 - R_2}{2} \sin 2\theta}{\Sigma \frac{R_1 - R_2}{2} \cos 2\theta} = \frac{\Sigma(R_1 - R_2) \sin 2\theta}{\Sigma(R_1 - R_2) \cos 2\theta} \quad \ldots\ldots(17)$$

which gives the value of Δ and thus locates the principal axes.

For walls, $R_2 = 0$, the formula reduces to

$$\tan 2\Delta = \frac{\Sigma R_1 \sin 2\theta}{\Sigma R_1 \cos 2\theta} \quad \ldots\ldots(18)$$

Distribution of forces

For a force applied through the center of rotation and along one of the principal axes, a pure translation will be produced along that axis with no translation perpendicular to it. The relative translation of the upper with respect to the lower horizontal diaphragm is given by Eq. 4 and 5 where ΣR_x and ΣR_y are the summation of rigidities of the elements along the two principal axes of the entire group.

Now considering each element separately, the forces along the principal axes of the element can be determined as follows. Referring to Fig. 5, the translation of Δ_x can be resolved into components of translation $\Delta_x \cos \phi$ along axis 1-1 and $\Delta_x \sin \phi$ along axis 2-2. The forces producing such components of translation are, evidently, $R_1 \Delta_x \cos \phi$ and $R_2 \Delta_x \sin \phi$, respectively. Similarly, for translation of Δ_y, the components of forces are $R_1 \Delta_y \sin \phi$ and $R_2 \Delta_y \cos \phi$. Hence the total components of forces along 1-1 and 2-2 are F_1' and F_2'—determined from Eq. 6 and 7.

When considering walls instead of columns, $R_2 = 0$ and $F_2' = 0$.

To determine the components of forces along the principal axes due to rotation, we must measure or compute the distances r_1 and r_2 to the principal axis of each element, calculate $J = \Sigma(R_1 r_1^2 + R_2 r_2^2)$, and apply Eq. 2 and 3. The total force components along 1-1 and 2-2 for each element are then F_1 and F_2 computed from Eq. 8 and 9. These forces can be combined vectorially into F.

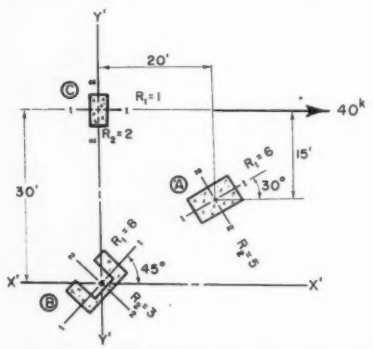


Fig. 7—Principal axes of skewed columns and rigidities along principal axes

Example 3 shows the solution for the general case—irregular columns at skew. For walls instead of columns, the solution is simplified because the rigidity about one principal axis is zero for each wall. If the skew walls or columns are symmetrically arranged about one axis, then the axis of symmetry is a principal axis of the group and the center of rigidity lies on that axis; thus the problem will be further simplified.

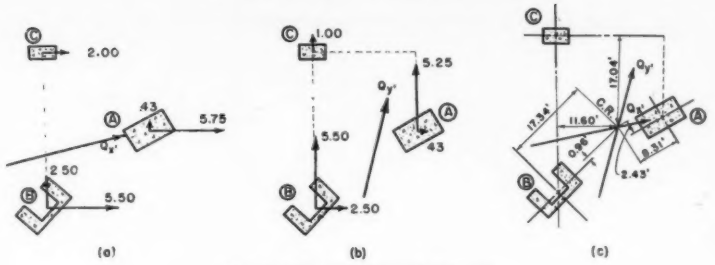


Fig. 8—Forces in skewed columns

Example 3

Three columns in a building are arranged skew to one another as shown in Fig. 7. The principal axes of the columns and the rigidities along the principal axes are as given. For a horizontal force of 40 kips acting on the group as shown, compute the forces acting on the columns.

Step 1—Choose an arbitrary pair of axes X'–X' and Y'–Y'. Determine the rigidities of the columns along X'–X' and Y'–Y' by the following formulas and list the computation as in Table 2.

$$R_x' = R_1 \cos^2 \theta + R_2 \sin^2 \theta$$
$$R_y' = R_1 \sin^2 \theta + R_2 \cos^2 \theta$$
$$R_{xy}' = (R_1 - R_2) \sin \theta \cos \theta$$

Step 2—Assume a unit translation along X'–X', the forces on the columns are shown in Fig. 8a and the resultant Q_x' located as shown. Then assume a unit translation along Y'–Y'. Forces are shown in Fig. 8b and the resultant Q_y' located as shown. The intersection of Q_x' and Q_y' is located as in Fig. 8c and is the center of rotation. Perpendicular distances from the center of rotation to the principal axes of the columns are measured and shown in Fig. 8c.

TABLE 2—RIGIDITY OF COLUMNS ALONG X'–X' AND Y'–Y'– AXES

Column	R_1	R_2	θ, deg	$\sin \theta$	$\cos \theta$	$\sin^2\theta$	$\cos^2\theta$	R_x'	R_y'	R_{xy}'
A	6	5	30	.500	.866	.250	.750	5.75	5.25	0.43
B	8	3	45	.707	.707	.500	.500	5.50	5.50	2.50
C	2	1	0	0	1	0	1	2.00	1.00	0

TABLE 3—PRINCIPAL AXES OF GROUP

Column	$R_1 - R_2$	θ, deg	2θ, deg.	$\sin^2\theta$	$\cos^2\theta$	$\frac{(R_1-R_2)}{\sin^2\theta}$	$\frac{(R_1-R_2)}{\cos^2\theta}$
A	6 − 5	30	60	.866	.500	0.87	0.50
B	8 − 3	45	90	1	0	5.00	0
C	2 − 1	0	0	0	1	0	1
					Σ	5.87	1.50

TABLE 4—J COMPUTED FROM VALUES OF r IN FIG. 8

Column	R_1	R_2	r_1	r_2	$R_1r_1^2$	$R_2r_2^2$	F_1''	F_2''
A	6	5	2.43	8.31	36	344	4.96 ↙	14.20 ↘
B	8	3	0.96	17.34	7	902	2.63 ↙	17.80 ↘
C	2	1	17.04	11.60	582	135	11.63 →	3.97 ↑
				Σ =	625	1381		

Step 3—Location of principal axes of group (Table 3).

$$\tan 2\Delta = \frac{\Sigma R \sin 2\Theta}{\Sigma R \cos 2\Theta} = \frac{5.87}{1.50} = 3.91$$

$$2\Delta = 75° 40' ; \quad \Delta = 37° 50'$$

Hence the principal axes for the group are located as X–X and Y–Y in Fig. 9.

Step 4—From values of r measured in Fig. 8, J is computed in Table 4.

$$J = \Sigma (R_1r_1^2 + R_2r_2^2) = 625 + 1381 = 2006$$

Step 5—The moment acting on the group is $M = 40 \times 17.16 = 686.4$ kip-ft

Forces on the elements due to moment are computed from Eq. 2 and 3 and listed in Table 4 as F_1'' and F_2''.

Step 6—Rigidities of the columns along the principal axes of the group, R_x and R_y, are computed as in Table 5. R_{xy} values are computed as a check; they should sum up zero along the principal axes.

The direct load is resolved into P_x and P_y. Δ_x and Δ_y are then computed by the following formulas:

$$P_x = 40 \cos (-37° 50') = 31.6 \text{ kips}$$
$$P_y = 40 \sin (-37° 50') = -24.5 \text{ kips}$$
$$\Delta_x = P_x/\Sigma R_x = 31.6/15.56 = 2.03$$
$$\Delta_y = P_y/\Sigma R_y = -24.5/9.47 = 2.59$$

F_1' and F_2' are computed from Eq. 6 and 7 and listed as in Table 6.

Step 7—F_1 and F_2 are obtained as the vectorial sum of F_1', F_1'' and F_2', F_2'', respectively. They are listed in Table 6. The resultant forces F are not desired and hence not computed.

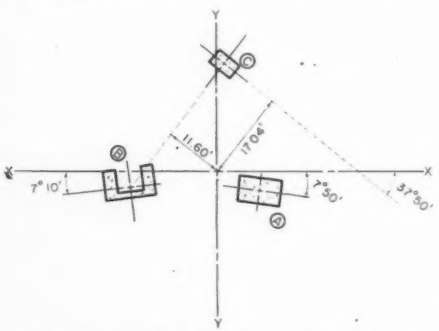


Fig. 9—Principal axes of group of columns

Simplification and Extension of the Method

While only fundamentals of the method are presented, it can be varied or simplified in certain cases, and extended to include other problems.

1. Since the resisting action of a wall against both direct load and torsion is independent of its location provided the wall remains on the same straight line (Fig. 10) it is evident that all

TABLE 5—RIGIDITIES OF COLUMNS ALONG X–X AND Y–Y AXES

Column	R_1	R_2	ϕ	sin ϕ	cos ϕ	$R_1 \cos^2 \phi + R_2 \sin^2 \phi = R_x$	$R_2 \cos^2 \phi + R_1 \sin^2 \phi = R_y$	$(R_1 - R_2)\sin \phi \cos \phi = R_{xy}$
A	6	5	7°50′	.136	.990	5.88 .10 5.98	4.90 .12 5.02	.13
B	8	3	−7°10′	−.125	.992	7.88 .05 7.93	2.95 .13 3.08	−.62
C	2	1	37°50′	.613	.790	1.25 .38 1.63	.62 .75 1.37	.49
					$\Sigma =$	15.54	9.47	0

TABLE 6—COMPUTATION OF FORCES DUE TO DIRECT LATERAL LOAD

Column	R_1	R_2	ϕ	$\Delta_x \cos\phi$	$\Delta_y \sin\phi$	F_1'	$\Delta_y \cos\phi$	$\Delta_x \sin\phi$	F_2'	F_1	F_2
A	6	5	− 7°50′	2.01	0.35	14.22 ↗	−2.56	−0.27	11.42 ↘	9.26 ↗	25.62 ↘
B	8	3	+ 7°10′	2.02	−0.32	13.52 ↗	−2.57	0.25	8.48 ↘	10.89 ↗	9.32 ↖
C	2	1	−37°50′	1.60	1.60	6.40 →	−2.04	−1.24	0.80 ↓	18.03 →	3.17 ↑

walls on a line can be combined and considered as one wall. In most buildings, there are only a few lines of walls, though there may be many walls in one line. This combination naturally simplifies the numerical computation.

2. The method can be readily applied to a combination of walls and columns because walls are simply columns with zero rigidity about one axis. The solution for walls is simpler than for odd-shaped columns. A column can sometimes be conveniently considered as two walls along its principal axes.

3. For earthquake force acting through the center of mass and variable in direction, the determination of maximum load or stress in a column or wall can be solved similarly to the example in Appendix 2. Although only round columns are illustrated in that example odd shaped columns or walls introduce no complications except that the stresses or forces must be considered along the principal axes and then combined if desired.

4. Forces acting on the vertical resisting elements having been determined, bending moments and shears existing in the horizontal diaphragm can be

Fig. 10—Walls A and B have the same effect on any group of resisting elements because they lie on one line

readily computed by statics. These serve as a basis for designing the horizontal diaphragms. However, if the horizontal diaphragms are relatively flexible so that their deflections are of the same order of magnitude as the vertical resisting elements, then this method must be modified. The solution considering elastic horizontal diaphragms will be more complicated, but the method presented here will still serve as the basis for such an investigation.

5. While this discussion is confined mainly to a concrete building, the theory and method are applicable to any building where the lateral force is distributed among elastic vertical elements through horizontal diaphragms.

CONCLUSIONS

This is believed to be the first general solution for distribution of lateral forces in a building story, including rectangular and odd-shaped columns at right and skew angles as well as walls at skew. The author recently had the opportunity to apply it to the design of several buildings with skew wings; and the analysis revealed that approximate methods yield radically different results. Analysis by this rigidity center method showed many weak spots in a building which would not have been detected otherwise.

Especially for an unconventional building layout, the location of the center of rigidity and the principal axes of the group of vertical resisting elements is not only an important problem but also an interesting and revealing one. The distribution of forces in proportion to the skew rigidities of the elements is also an important basic understanding which the engineer should be able to grasp and apply.

ACKNOWLEDGMENT

The author is grateful to Stanley C. King, of Ellison and King, Consulting Structural Engineers, San Francisco, for his interest and suggestions concerning the paper. It was while working with Mr. King on lateral force analysis of several skew buildings that the method here presented was first developed and applied.

APPENDIX 1
FORMULAS FOR RIGIDITIES OF VERTICAL ELEMENTS

Notation

Terms in the following are those commonly employed in moment distribution and it is assumed that readers are already familiar with the method.

For an element or member AB,

R = rigidity of AB
E = modulus of elasticity of AB
I = moment of inertia of AB in direction of bending
L = length of member AB
K = stiffness of AB for uniform I; K_a, K_b = stiffness for ends A and B in the case of varying I; K_{am}, K_{bm} = modified stiffness for ends A and B, considering partial fixation of the ends. ΣK_{am}, ΣK_{bm} = summation of the modified stiffnesses of all members meeting at joint A and B, respectively.

C = carry-over factor of AB; C_a, C_b = carry-over factor from A to B, from B to A, respectively; C_{am}, C_{bm} = modified carry-over factor from A to B, from B to A.
A = cross-sectional area of member AB
E_s = modulus of rigidity of material
Δ_s = displacement of A with respect to B, due to shear effect, for unit load
Δ_m = displacement of A with respect to B, due to moment effect, for unit load

Neglecting effect of shear deformation

(1) Members with both ends fixed (Fig. 11a).

Uniform I, $\quad R = 12EI/L^3 = 3K/L^2$

Variable I, $\quad R = \frac{1}{L^2}[K_a(1+C_a) + K_b(1+C_b)]$

Fig. 11a—Member with both ends fixed

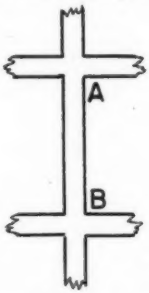
(2) Members with one end fixed and one end hinged (Fig. 11b).

Uniform I, $\quad R = 3EI/L^3 = 3K/4L^2$

Varying I, $\quad R = \frac{K_b}{L^2}(1 - C_aC_b)$

Fig. 11b—Member with one end fixed and one end hinged

(3) Members with partially restrained ends (Fig. 11c).*



Uniform I, $R = \frac{1.5}{L^2}\left[K_a\left(1 - \frac{K_{am}}{\Sigma K_{am}}(1 + C_{am})\right) + K_b\left(1 - \frac{K_{bm}}{\Sigma K_{bm}}(1 + C_{bm})\right)\right]$

Varying I, $R = \frac{1}{L^2}\left[K_a(1 + C_a)\left(1 - \frac{K_{am}}{\Sigma K_{am}}(1+C_{am})\right) + K_b(1 + C_b)\left(1 - \frac{K_{bm}}{\Sigma K_{bm}}(1 + C_{bm})\right)\right]$

Fig. 11c—Member with partially restrained ends

These two formulas do not take into account the bending of the adjacent members due to lateral forces acting in their own stories. If this bending is to be considered, the exact determination of rigidities for members with partially restrained ends will be further complicated.

*See Lin, T. Y., "A Direct Method of Moment Distribution," *Trans.*, ASCE, 1937 p. 561, for computation of modified stiffnesses and modified carry-over factors.

Considering effect of shearing deformation

$\Delta_s = 1.2L/E_s A$, assuming beam shear distribution in a rectangular section
$\Delta_m = 1/R_m$, where R_m is the rigidity obtained above considering moment only
$R = 1/(\Delta_s + \Delta_m)$, considering both moment and shear effects.

The significance of shear deformation can be appreciated when we consider, for example, that a square panel of concrete wall with partially restrained ends will have a shearing deformation about equal to its moment deformation, both along the length of the wall.

APPENDIX 2
EXAMPLE 4—DIRECTION OF FORCE FOR MAXIMUM LOAD ON AN ELEMENT

Suppose the building in Example 1 is subjected to an earthquake load which can occur in any direction. The load is given as 30 kips passing through the center of mass located as shown. It is required to determine the direction of earthquake producing the greatest load on column A.

Let β be the angle between the force and the X–X axis (Fig. 12). Then the moment M about the center of rotation will be 6 x 30 cos β ; $P_x = 30 \cos \beta$; $P_y = 30 \sin \beta$.

Using values of R's and J as computed in Example 1, we have

$$F_1'' = \frac{M_y}{J} R = \frac{180 \cos \beta \times 20}{1830} 1 = 1.97 \cos \beta$$

$$F_2'' = \frac{M_x}{J} R = \frac{180 \cos \beta \times 15}{1830} 1 = 1.48 \cos \beta$$

$$\Delta_x = \frac{30 \cos \beta}{6} = 5 \cos \beta \, , \; \Delta_y = \frac{30 \sin \beta}{6} = 5 \sin \beta$$

$$F_1' = R_1 (\Delta_x \cos \phi + \Delta_y \sin \phi) = 1 (5 \cos \beta) = 5 \cos \beta$$

$$F_2' = R_2 (\Delta_y \cos \phi - \Delta_x \sin \phi) = 1 (5 \sin \beta) = 5 \sin \beta$$

$$F = \sqrt{(F_1' + F_1'')^2 + (F_2' - F_2'')^2}$$

$$= \sqrt{(1.97 + 5)^2 \cos^2 \beta + (- 1.48 \cos \beta + 5 \sin \beta)^2}$$

To determine β for maximum F, we square both sides, differentiate, and equate to zero:

$$\frac{d F_2}{d \beta} = \frac{d}{d \beta} \left[6.97^2 \cos^2 \beta + (- 1.48 \cos \beta + 5 \sin \beta)^2 \right] = 0$$

$$- 6.97^2 \times 2 \cos \beta \sin \beta + 2 (- 1.48 \cos \beta + 5 \sin \beta) (1.48 \sin \beta + 5 \cos \beta) = 0$$

$$\cos \beta \sin \beta [- 96.7 - 4.35 + 50] - 14.8 \cos^2 \beta + 14.8 \sin^2 \beta = 0$$

$$- 3.44 \cos \beta \sin \beta - \cos^2 \beta + \sin^2 \beta = 0$$

$$- 1.72 \sin 2 \beta - \cos 2 \beta = 0$$

$$\tan 2 \beta = \frac{- 1}{1.72} = - .582$$

$$2 \beta = - 30° \, 12'$$

$$\beta = - 15° \; 6'$$

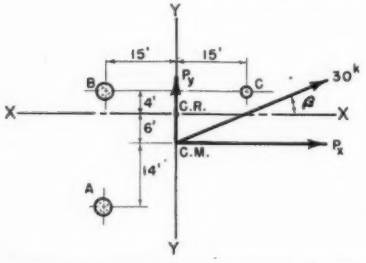


Fig. 12—Earthquake load of variable direction superimposed on conditions n Fig. 3

Which means that for the force acting at $\beta = -15° \, 06'$, we will get the maximum load on column A. The magnitude of F can be obtained by substituting β in the above formula for F, obtaining $F = 6.74$ kips, the maximum for column A.

Title No. 48-20

Air Entrainment and Resistance to Freezing and Thawing

By E. W. SCRIPTURE, JR.†, S. W. BENEDICT†, and F. J. LITWINOWICZ†

SYNOPSIS

Investigations were undertaken to determine the suitability of various air-entraining agents for use in concrete, the relative effects of slow and rapid cycles of freezing and thawing, and the optimum range of air contents for concrete. With normal air-entraining agents the resistance to freezing and thawing varies mainly with the air content, not with the particular agent used. A rapid freezing and thawing cycle is considerably more destructive than a slow cycle, and abnormal results appear to be produced in some cases by a very fast cycle. With increasing entrained air resistance to freezing and thawing increases to a maximum and thereafter no further benefit appears to be secured. The optimum amount seems to be about 2½ to 3 percent *added* entrained air.

INTRODUCTION

The use of air-entraining concrete to secure increased resistance to freezing and thawing and to minimize scaling of pavements has now become widespread. While it is quite generally agreed that entrainment of a suitable amount of air in concrete will increase resistance to freezing and thawing, there is considerably less agreement on some other aspects of the problem.

Although there is a substantial body of literature on air-entraining agents, there does not appear to be any report of work indicating what air-entraining agents are and are not acceptable, except for three or four approved by ASTM Committee C-1 for intergrinding with cement, whereas there are at least thirty others which might be used either interground or added at the mixer, which have not been tested under the procedure (now abolished) of Committee C-1 for interground additions. Committee C-9 of ASTM has recently set up a tentative method of test of air-entraining agents and a specification for the acceptability of air-entraining agents for addition at the mixer, and C-1 has developed such a specification for interground agents.

One of the criteria of acceptability of air-entraining agents is the resistance of a standard concrete mix to freezing and thawing. There is, however, no generally accepted freezing and thawing test. Practically every apparatus for making determinations of resistance to freezing and thawing seems to

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differ from all others. Consequently, an air-entraining agent found acceptable by one method of test might not be acceptable when another method was used. Similarly, the type of freezing and thawing test may affect the order of merit in which admixtures might be arranged.

Specifications for the amount of entrained air desired in concrete vary widely. In a report of a survey made by AASHO*, answered by the Highway Departments of 39 states and the District of Columbia, the minimum specified air content ranges from 2 to 3 percent and the maximum from 3½ to 7 percent. These agencies listed the most desirable range of air contents as a minimum of from 2 to 3 percent and maximum from 3 to 10 percent. Between the limits 2 and 10 percent there is no very great agreement on the most desirable entrained air content.

A test program was therefore undertaken in an attempt to throw some light on the relative effects of different air-entraining agents, the relative effects of a fast and a slow cycle freezing and thawing test, and the optimum air content.

TEST PROGRAM

Comparison of admixtures

The effects of different air-entraining agents were investigated in three series of tests covering six different air-entraining agents. In the first series three air-entraining agents were used and compared with concrete containing no air-entraining agent. These air-entraining agents were Admixture A—a petroleum hydrocarbon insoluble fraction of a coal-tar hydrocarbon extract of pine wood (neutralized), Admixture C—a calcium lignosulfonate cement dispersing agent, and Admixture D—a combination of the calcium lignosulfonate cement dispersing agent with lauryl sodium sulfate (this air-entraining agent was diluted with a small amount of fly ash before addition to the mix.) These tests were made with equal cement contents with the same sand-total aggregate ratio for all mixes and with the sand-total aggregate ratio adjusted where the air-entraining agents were used, and also with cement contents varied to produce equal strengths with both constant sand-total aggregate ratio and with adjusted sand-total aggregate ratio. In the second series Admixtures A and C were again used and also Admixture B—a triethanolamine salt of a sulfonated hydrocarbon, and Admixture E—a sulfonated alkyl aryl surface active agent. Admixtures A, B, and C are the air-entraining agents most widely used in concrete. This series consisted of a single set of mixes with equal cement factors, with approximately the same air contents for all mixes except that with no addition. In the third series four admixtures were used, Admixtures A, B, and C and Admixture F—a surface active agent of unknown composition not commercially used as an air-entraining agent for concrete. This series consisted of a single set of mixes all at the same cement content and approximately the same air content, and made in accordance with the ASTM Tentative Method for Testing Air Entraining Admixtures C 233-49T.

Rate of freezing and thawing

Two series of tests were made to compare a slow freezing and thawing cycle, one cycle per day, with a fast freezing and thawing cycle, twelve cycles per day. The first of these series was identical with the third series described above in the program on different admixtures. In the second series the admixtures used were C, described above, and Admixture G, the calcium lignosulfonate cement dispersing agent with the addition of a surface active agent to promote some additional air entrainment. The air-entrain-

*"Specifications on Air Entrainment," *Concrete*, Jan. 1948.

ing concretes were compared with a concrete mix to which no air-entraining agent was added and the comparisons were made at approximately equal cement factors and at reduced cement factors with the air-entraining agents.

The slow cycle of freezing and thawing was used for the part of the program devoted to evaluation of the effects of different air-entraining agents. The fast cycle of freezing and thawing was used in two of the series to evaluate different air-entraining agents and for evaluation of the relation between resistance to freezing and thawing and air content. Obviously both types were used for the comparison between the effects of fast and slow cycles.

Varied air content

The third part of the program consisted of two series of tests in which air contents were varied from as low as could be obtained to approximately 10 percent. In each series mixes were made with a nominal 6-sack and with a nominal 4½-sack cement factor, using two different cements in each case. To secure the desired range of air contents several different additions were used. These included an air-detraining agent to reduce air content to a minimum in some cases, the calcium lignosulfonate cement dispersing agent to give normal air contents, and the calcium lignosulfonate cement dispersing agent combined with an added surface active material to produce the higher air contents. The surface active material was also used by itself for some of the intermediate air contents. This procedure was adopted because previous work had indicated that the method of entraining air, at least with the materials involved, did not materially affect the resistance of the concrete to freezing and thawing.

General conditions

In the test to evaluate different air-entraining agents a nominal slump of 3 in. was used in the first and third series, a nominal 4½-in. slump in the second. In the program to compare fast and slow cycles of freezing and thawing a nominal 3-in. slump was used. For the program on the variation of resistance to freezing and thawing with entrained air contents, a nominal slump of 4 in. was used throughout.

Type I cement was used for all mixes except the second series of tests to compare fast and slow cycles, for which Type III cement was used. The surface areas of the cements, the amounts of air entrained by them in the ASTM mortar test, and the physical constants of the sand and limestone aggregate used are given in Table 1.

The amount of Admixture A used varied from .007 to .01 percent, that of Admixture B from .01 to .012 percent, and that of Admixtures C and G was 0.5 percent by weight of the cement. Admixtures D, E, and F were used in proportions 1.0, .002, and .038 percent, respectively. In evaluating the effects of variable air contents the amounts of admixture used were varied to produce the desired air contents.

TEST PROCEDURE

The concrete mixes were made in batches of approximately 2 cu ft, mixed 3 minutes in a 2½ cu ft tilting-drum concrete mixer. Slumps were measured and 6 x 12-in. cylinders were made, cured and broken to determine compressive strengths using appropriate ASTM standard methods. Air contents were calculated gravimetrically and determined immediately after mixing by the pressure method with a Klein-Walker air meter or the Washington air meter. Determinations of entrained air in standard mortar mixes were made in accordance with ASTM Method Designation C 185-47T. Determinations of surface area were made with the Blaine air permeability apparatus and converted by factor to Wagner values.

The slow cycle of freezing and thawing was done in a freezing chamber kept at a temperature of 0 to 5 F. Every 24 hours the specimens were removed from the freezing chamber and thawed immersed in a tank in running tap water for 7 hours at approximately 60 F. The fast cycle of freezing and thawing in water was carried out in an

TABLE 1—PHYSICAL CONSTANTS OF MATERIALS

Sieve size	Air-entraining agents			Cycle Series II	Air content Series I	Air content Series II
	Series I	Series II	Series III			
			Fine aggregate—Percent retained			
4	5	5	2	2	1	2
8	14	14	13	11	7	12
16	30	30	27	24	19	27
30	49	49	45	45	40	46
50	88	88	82	85	85	83
100	99	99	99	98	99	98
Pan	1	1	1	2	1	2
FM	2.83	2.83	2.78	2.65	2.51	2.68
Sp.gr.	2.60	2.60	2.60	2.60	2.60	2.60
			Coarse aggregate—Percent retained			
1½ in.	0	0	0	3	0	0
1 in.	58	58	0	58	0	0
¾ in.	91	91		90	17	11
½ in.	96	96	50*	96	78	58
⅜ in.	97	97		97	97	88
¼ in.	98	98	0	98	99	99
Pan	2	2		2	1	1
FM	7.86	7.86		7.80	7.13	6.88
Sp. gr.	2.76	2.76		2.82	2.66	2.64
			Cement			
Percent air	6.4	5.9	6.4 5.4 7.6	7.6	7.0	8.2
Surface area	1670	1650	1740 1670 1620	2520	1680	1820

*Prepared in accordance with ASTM Designation C 233-49T

apparatus described in Appendix 1. The cycle consisted of freezing for 80 minutes, in which the temperature was reduced to 5 F at the center of the specimen and approximately − 10 F at the surface, and thawing for 40 minutes, in which the temperature was raised to 60 F at the center and 75 F at the surface. Necessarily the difference between the fast and slow cycles is not solely one of time, since to produce more rapid freezing and thawing larger temperature differentials are required. The temperatures in both the fast cycle and slow cycle apparatus were automatically controlled. In the fast cycle apparatus the time cycle was automatically controlled and the temperature changes were recorded.

The specimens used for all freezing and thawing tests were 3 x 3½ x 15-in. beams, molded in accordance with ASTM Designation C 192-49, cured 27 days in the moist closet, and soaked 24 hours in water before starting the freezing and thawing tests. Three specimens were used for each mix, and the figures given represent the averages of the three specimens. Deterioration in the freezing and thawing cycles was determined by measurement of the dynamic modulus of elasticity by the sonic method. Durability factors were calculated by the formula $DF = PN/200$

where

DF = durability factor

P = relative dynamic modulus of elasticity in percentage of the dynamic modulus of elasticity at zero cycles

N = number of cycles at which P reaches 70 percent, or 200 if P does not reach 70 percent prior to 200 cycles.

TEST RESULTS

Comparison of admixtures

Results of tests made with admixtures A, C, and D are given in Table 2. For the two groups made at equal cement contents the water-cement ratios

TABLE 2—EFFECT OF AIR-ENTRAINING AGENTS ON STRENGTH AND DURABILITY OF CONCRETE, SERIES I, NOMINAL SLUMP 3 IN.

Addition	Cement factor, sacks per cu yd	W/C, gal. per sack	Percent air	Compr. Str., 6 mo., psi	Percent loss in dynamic E, After cycles							DF
					35	50	75	100	150	200	250	
Sand/Total aggregate—40/60												
None	6.02	6.04	0.2	6870	19	29	46	—	—	—	—	18
A	5.93	5.66	3.7	6270	2	4	8	10	19	29	—	71
C	5.98	5.31	3.7	7610	3	7	9	9	17	30	—	70
D	5.97	5.35	3.5	7510	3	3	3	4	8	16	24	84
Sand/Total aggregate—36/64												
A	5.95	5.61	3.4	6255	1	3	5	9	22	31	—	69
C	6.00	5.20	3.5	7460	2	3	4	5	12	24	42	76
D	5.98	5.16	3.6	7265	1	2	3	6	13	21	25	79
Sand/Total aggregate—40/60												
A	6.34	5.35	3.3	6880	1	4	5	12	23	32	—	68
C	5.21	5.80	3.6	7350	6	9	16	22	39	—	—	36
D	5.51	5.60	3.7	6920	4	5	6	17	35	—	—	59
Sand/Total aggregate—36/64												
A	6.35	5.35	3.3	6820	4	6	6	11	26	46	—	52
C	5.22	5.72	3.5	7660	8	9	9	13	20	31	—	69
D	5.53	5.58	3.5	6710	5	8	12	22	37	—	—	47

are lower with Admixtures C and D than with no addition and with Admixture A; compressive strengths are correspondingly higher. With Admixture A, although the water-cement ratio is lower than that of the mix with no addition, there is a reduction in strength. The air contents with all three of the admixtures are approximately the same but they are probably lower than the true air contents since they were determined by the gravimetric method instead of by the pressure method. In these two groups the resistance to freezing and thawing is approximately the same for the three admixtures, although in one group C and D show slightly better resistance and in the other group D shows somewhat better resistance, with A and C approximately the same. Where the sand-total aggregate ratio is decreased to compensate for the entrained air, the water reductions are somewhat greater, but otherwise the effects are not large.

Two groups were run with adjustments in the cement factor intended to produce approximately equal strengths. This result was attained with Admixtures A and D, but C still gave somewhat higher strengths. Again, the air contents were approximately equal for the three admixtures. Resistance to freezing and thawing was approximately the same for all three admixtures: in one group A showed slightly greater resistance than C and D, whereas in the other group C was slightly more resistant than A and D slightly less. It would appear that the improved resistance to freezing and thawing secured by entrained air, for a given air content, is approximately the same regardless of the manner in which the air is entrained, at least insofar as Admixtures A, C, and D are concerned.

TABLE 3—RELATIVE RESISTANCE TO FREEZING AND THAWING OF AIR-ENTRAINING CONCRETES, SERIES II, NOMINAL SLUMP 4½ IN.

Addition	Cement factor, sacks per cu yd*	W/C, gal. per sack	Percent air	Compr. Str., 28 days, psi	Percent loss in dynamic *E*, After cycles								DF
					25	50	75	100	125	150	175	200	
None	5.48	6.57	1.8	3800	56	79	87	100	—	—	—	—	6
A	5.55	6.01	4.6	4120	4	7	6	7	8	10	16	21	79
B	5.57	6.01	4.3	3750	4	6	9	9	10	13	17	24	76
C	5.54	5.80	4.7	4050	5	8	8	8	9	16	15	17	83
E	5.54	5.97	4.8	3540	2	5	7	9	9	11	19	20	80

*Nominal cement factor 5½ sacks per cu yd.

Resistance to freezing and thawing

The second series of tests (Table 3), was designed to compare the relative resistance to freezing and thawing of air-entraining concretes made with three of the most widely used admixtures, A, B, and C, and Admixture E. A mix with a nominal cement factor of 5½ sacks per cu yd was used. Values in Table 3 are the averages for the two separate series run. All four admixtures had approximately the same resistance to freezing and thawing, which again leads to the conclusion that the determining factor is the amount of air entrained and not the air-entraining agent. The values for loss in dynamic modulus for this second series are shown in Fig. 1 and are typical of the comparisons between different air-entraining agents. With no addition the dynamic modulus falls off very rapidly, whereas with the four admixtures resistance to freezing and thawing is greatly increased. A single curve represents the behavior of all four admixtures within the limits of accuracy of freezing and thawing tests.

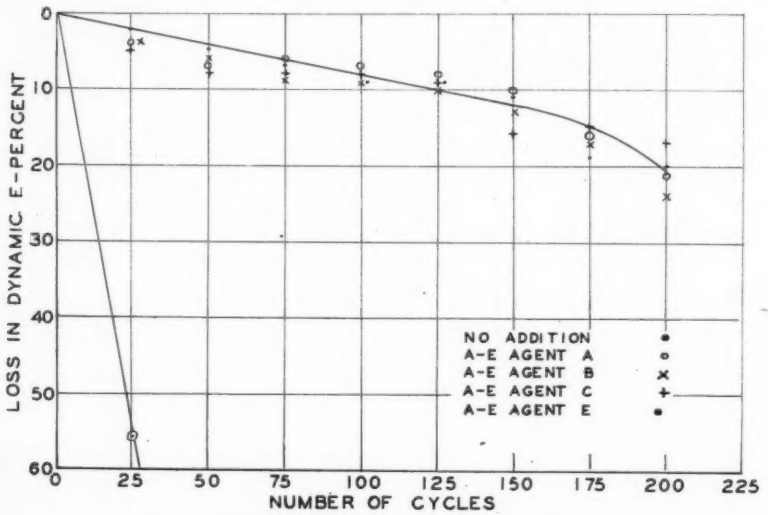

Fig. 1—Loss in dynamic E in freezing and thawing, Series II

TABLE 4—COMPARATIVE DATA ON FAST AND SLOW FREEZING OF CONCRETES CONTAINING VARIOUS AIR-ENTRAINING AGENTS, SERIES I, NOMINAL SLUMP 3 IN.

Addition	Cement factor, sacks per cu yd*	W/C, gal. per sack	Percent air	Percent loss in dynamic E, After cycles											DF
				25	50	75	100	125	150	175	200	225	250	275	
Slow cycle															
A	5.04	6.51	4.2	0	0	+2	2	8	15	16	23	28	33	45	77
B	5.01	6.53	4.7	0	0	+2	11	17	26	28	37	48	49	61	63
C	5.02	6.24	4.2	+1	+2	+2	2	12	21	19	28	37	40	52	72
F	5.04	6.30	4.8	0	0	0	1	6	15	19	29	33	35	49	71
Fast cycle															
A				4	7	11	14	13	24	31	39				60
B				5	7	8	13	11	24	32	39				59
C				8	8	16	14	10	24	30	30				70
F				42	61	94	100	—	—	—	—				23

*Nominal cement factor 5 sacks per cu yd.

The third series of tests (Table 4), was similar to the second series but substituted a surface active agent of unknown composition (Admixture F) for the sulfonated alkyl aryl compound of the previous series (Admixture E). Also, these tests were made strictly in accordance with the Tentative Method for Evaluation of Air Entraining Admixtures in Concrete, ASTM C 233-49T, using a blend of three cements and with all the mixes having approximately the same air contents. With the slow cycle of freezing and thawing all four of the admixtures had approximately the same resistance to freezing and thawing, which confirms the conclusion reached on the basis of the results of the two preceding series. That this is not always the case will be seen in the comparison between the fast and slow cycles of freezing and thawing which follows.

Results of two test series comparing the slow cycle of freezing and thawing with the fast cycle are given in Tables 4 and 5. In the first series (Table 4)

TABLE 5—RESISTANCE TO FREEZING AND THAWING AND TYPE OF CYCLE, SERIES II, NOMINAL SLUMP 3 IN.

Addition	Cement factor, sacks per cu yd	W/C, gal. per sack	Percent air	Compr. Str., 28 days psi	Percent loss in dynamic E, After cycles								DF
					25	50	75	100	125	150	175	200	
Fast cycle													
None	5.94	5.43	2.5	5755	5	13	18	21	22	25	27	36	64
C	5.91	4.82	5.1	6595*	3	9	13	16	15	21	22	27	73
G	5.82	4.72	6.8	6035*	6	7	12	16	16	19	20	24	76
C	5.04	5.23	5.8	6570	2	10	14	14	18	20	21	24	76
G	4.97	5.25	6.8	6935	2	4	9	11	13	15	17	19	81
Slow cycle													
None					-4	0	+1	+3	0	+1	4	1	99
C					0	1	1	+1	0	+1	5	0	100
G					0	0	2	0	0	1	6	4	96
C					0	+1	+3	0	+1	0	5	0	100
G					+2	0	+2	+3	+2	0	6	1	99

*Lower than actual strength since one or more cylinders of this group exceeded the capacity of the testing machine.

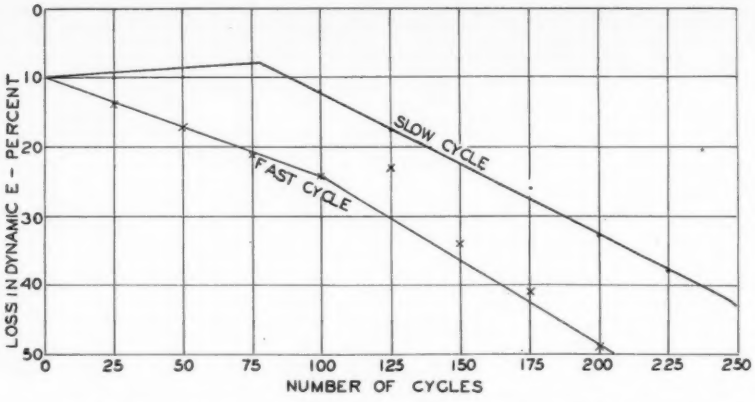

Fig. 2—Fast freezing and thawing increases loss of strength rate, Series I, Admixture A

resistance to freezing and thawing in the slow cycle is approximately the same for all four admixtures. Except for Admixture F, this is also true of the fast cycle which is definitely more destructive than the slow cycle. One curious result in this series is that Admixture F deteriorated very rapidly in the fast cycle, much more rapidly than the other three admixtures, but this effect was not apparent in the slow cycle. It seems, therefore, that there may be cases where, even at the same air content, different admixtures may produce different resistances to freezing and thawing under some freezing and thawing conditions.

In the second series (Table 5) very high strengths were secured with high early strength cement and resistance to freezing and thawing was exceptionally high for the mix with no addition. The increased resistance to freezing and thawing secured with air entrainment was not, therefore, as great as is usually the case. A definite improvement in the fast cycle was, however, secured with the air-entraining concretes and resistance to freezing and thawing of these mixes was approximately the same for all four admixtures. In the slow cycle, at the end of 200 cycles, practically no deterioration had occurred in any of the mixes. It is, however, again evident that the fast cycle is more destructive than the slow cycle.

Fig. 2 shows a comparison between the slow and fast cycles for Admixture A in the first series, and Fig. 3 shows the results for No Addition in the second series.

Varied air content

The results of the two series of tests made at different times with varying air contents are given in Tables 6 and 7. It is obvious in both cases that the lean mix deteriorates relatively more rapidly than the rich mix. In both series resistance to freezing and thawing increases with increasing air content

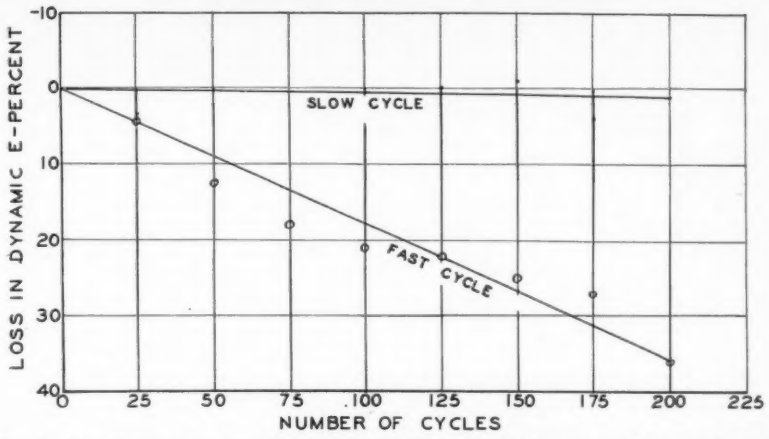

Fig. 3—Non-air-entraining concrete loses strength faster with fast freezing and thawing, Series II

up to a certain point, and beyond that point there is little or no added resistance to freezing and thawing.

Air contents in the first series were 0.9 and 1.9 percent with no addition, respectively, for the nominal 6-sack and the nominal 4½-sack mixes. In the second series the corresponding percentages were 2.3 and 2.5 percent; with the cement used air contents could not be reduced further. If now the concept is introduced of added air entrainment rather than total air content, a greater parallelism between the two series appears. In the first series there is no substantial increase in resistance to freezing and thawing with added air

TABLE 6—AIR CONTENT AND RESISTANCE TO FREEZING AND THAWING, SERIES I, NOMINAL SLUMP 4 IN.

Percent air	Additional entrained air, percent*	Cement factor, sacks per cu yd	W/C, gal. per sack	Compr. str., 28 days, psi	Percent loss in dynamic E, after cycles								DF
					25	50	75	100	125	150	175	200	
Nominal—6-sack mix													
0.9	—	6.05	6.05	5220	66	75	—	—	—	—	—	—	4
2.0	1.1	6.02	5.87	5750	14	16	18	28	32	44	52	59	36
2.8	1.9	5.98	5.83	5340	9	10	13	14	16	17	22	28	72
3.9	3.0	6.05	5.02	6600	9	13	13	15	18	23	28	35	63
4.9	4.0	5.98	5.02	6610	6	9	13	14	15	22	30	38	61
6.4	5.5	5.91	4.94	5920	7	11	13	14	15	21	26	32	65
7.7	6.8	5.85	4.82	5640	10	12	15	18	19	22	28	32	63
10.2	9.1	5.70	4.83	4580	6	8	12	13	14	19	25	31	66
Nominal 4½-sack mix													
1.9	—	4.40	8.76	3110	49	84	—	—	—	—	—	—	3
2.9	1.0	4.36	8.75	3080	11	40	—	—	—	—	—	—	4
4.3	2.4	4.32	8.50	2760	10	14	22	34	39	—	—	—	26
5.2	3.3	4.29	8.39	2780	4	14	24	31	36	—	—	—	30
5.7	3.8	4.34	7.67	3750	7	12	16	25	31	—	—	—	46
7.8	5.9	4.27	7.19	3630	11	16	26	30	34	—	—	—	49
10.0	8.1	4.17	7.39	2830	10	15	20	28	31	—	—	—	44

*Air content of mix less air content of mix with lowest air content.

TABLE 7—AIR CONTENT AND RESISTANCE TO FREEZING AND THAWING, SERIES II, NOMINAL SLUMP 4 IN.

Percent air	Additional entrained air, percent*	Cement factor, sacks per cu yd	W/C, gal. per sack	Compr. str., 28 days, psi	Percent loss in dynamic E, after cycles									DF
					10	25	50	75	100	125	150	175	200	
Nominal 6-sack mix														
2.3	—	6.23	5.93	5500	—	28	81	—	—	—	—	—	—	9
2.6	0.3	6.23	5.60	5500	—	11	29	36	48	68	—	—	—	18
2.9	0.6	6.28	5.25	6050	—	15	37	56	80	—	—	—	—	16
3.8	1.5	6.26	5.09	6740	—	10	17	22	41	47	60	—	—	29
4.6	2.3	6.24	4.91	6100	—	9	10	15	28	40	36	40	50	36
6.2	3.9	6.04	5.38	5550	—	10	13	16	17	19	28	34	35	54
7.7	4.4	6.02	5.01	5600	—	9	10	11	12	17	25	29	34	62
10.0	7.7	5.92	4.80	5040	—	9	10	12	15	21	23	30	35	61
Nominal 4½-sack mix														
2.5	—	4.48	8.10	3430	44	—	—	—	—	—	—	—	—	3
2.8	0.3	4.49	7.82	3430	24	44	—	—	—	—	—	—	—	4
4.3	1.8	4.43	7.86	3020	6	16	23	31	41	—	—	—	—	26
5.0	2.5	4.42	7.60	2950	6	10	16	20	35	38	—	—	—	30
6.4	3.9	4.35	7.59	3040	6	10	13	14	21	27	33	—	—	46
7.8	5.3	4.40	6.48	3840	5	7	12	19	17	22	31	—	—	49
10.5	8.0	4.26	6.55	2970	7	13	15	18	21	30	38	—	—	44

*Air content of mix less air content of mix with lowest air content.

over about 2 to 2½ percent. Similarly, in the second series there is no substantial increase in resistance to freezing and thawing with added entrained air over about 2½ to 3 percent. On the basis of these rather limited tests it may be inferred that increasing entrained air more than 2½ to 3 percent over the air content of the plain mix does increase resistance to freezing and thawing. A corollary of this is that the air content of the mix secured without air-entraining agent added does not contribute to freezing and thawing resistance. For example, the 6-sack mix with 2.3 percent air is no more resistant than the 6-sack mix with 0.9 percent air, and the 4½-sack mix with 2.5 percent air is actually less resistant than the corresponding mix with 1.9 percent. Apparently air introduced into or retained in the mix without air-entraining agent added is in a different form from that of the air entrained by an air-entraining agent.

Fig. 4 shows the durability factors for the mixes of the two series. For the 4½-sack mixes the agreement between the two series is excellent and the points are best represented by a single curve. For the 6-sack series there is somewhat greater scattering of results but the general trend is the same for both series. These curves are based on added air contents, not total air contents, which yields greater concordance between the two series: based on total airs there is a much greater difference between Series I and Series II, especially in the 6-sack mixes, again indicating that air in the mix with no addition differs from air entrained with air-entraining admixtures.

In both these series some anomalies in the strength results occur. This is because air contents were secured by different means. For example, the fourth mix, nominal 6-sack in the first series, introduces the cement dispersing agent calcium lignosulfonate, and the trend toward decreased strength with

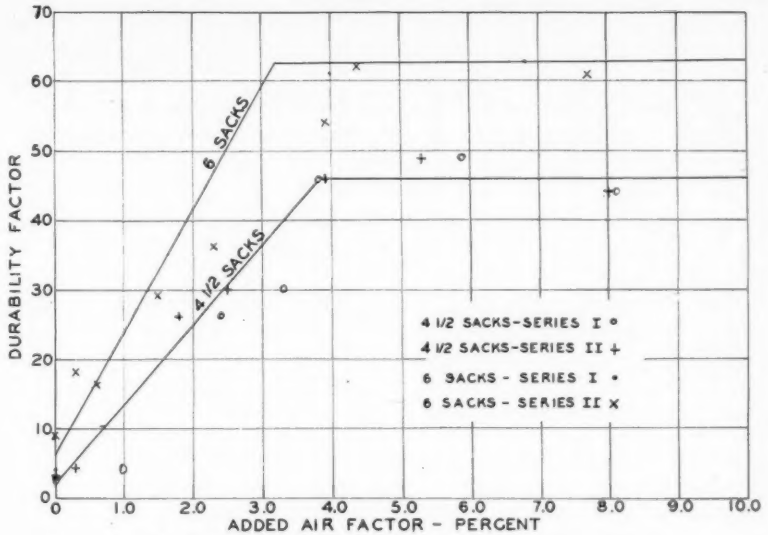

Fig. 4—Added air content and resistance to freezing and thawing

increased air content is reversed at this point. A similar reversal occurs in the fifth mix of the nominal 4½-sack mixes in the first series. Normally, however, it can be anticipated that strength will decrease with increasing air entrainment for a given air-entraining agent.* These different methods of securing the desired air contents in these two series were used because previous work indicated that the method of entraining air was not a determining factor in resistance to freezing and thawing, and it would otherwise have been difficult to vary the amount of air entrained.

The results indicate that to avoid loss in strength, air entrainment should be limited to the minimum which will give maximum resistance to freezing and thawing. As far as these test results go, this minimum appears to be about 2½ to 3 percent additional air, resulting in a total air content of 3½ to 4½ percent for normal mixes which could be expected to have air contents of 1 to 1½ percent without air-entraining agent.

CONCLUSIONS

1. With normal air-entraining agents resistance to freezing and thawing varies mainly with air content, not with the particular air-entraining agent used. With some unusual air-entraining agents, under unusual conditions such as very rapid cycles of freezing and thawing, this may not be the case.

2. Rapid freezing and thawing is considerably more destructive than a slow cycle. A very fast cycle may produce abnormal results in certain cases.

*Gonnerman, H. F., "Tests of Concretes Containing Air-Entraining Portland Cements or Air-Entraining Materials Added to Batch at Mixer," ACI JOURNAL, June 1944, *Proc.* V. 40, p. 477.

3. Resistance to freezing and thawing, other things being equal, varies with the air content of the concrete mix. This variation is more directly related to the *added* entrained air than to the total air content of the mix. Based on the data obtained in this investigation, with increasing entrained air resistance to freezing and thawing increases to a maximum and thereafter remains substantially constant. This maximum appears to be about 2½ to 3 percent *added* entrained air.

4. Strength and probably permeability play a part in resistance to freezing and thawing, as witness the greater resistance of a 6-sack mix compared with a 4½-sack mix.

5. Air contents in excess of the minimum which will produce maximum resistance to freezing and thawing are undesirable due to the reduction in strength with increasing air content.

APPENDIX 1

EQUIPMENT FOR ACCELERATED FREEZING AND THAWING TESTS ON CONCRETE

Refrigerating unit—The refrigerating unit consists of a compressor operated by a 3 hp motor and is manufactured by the Universal Cooler Co., Marion, Ohio, Model No. W-200 FS. The unit takes a 9-lb charge of Freon. The system has a pressure-reducing valve on the cooling water line feeding the compressor. A heat exchanger is installed in the gas line. A solenoid switch connected to both the heater and compressor circuit is operated by a Flexopulse automatic timer manufactured by the Eagle Signal Corp., Moline, Ill., which can be set for the complete cycle of freezing and thawing of any duration up to two hours.

Freezing cabinet—This equipment consists of a two-door cabinet freezer, outside dimensions: 5 ft high, 4 ft wide, and 28 in. deep. The freezing and thawing compartment is 50 in. wide, 39 in. high, and 17 in. deep and contains 8 plates through which the refrigerant circulates. Four of these plates serve as shelves holding the pans containing the specimens and the two upper and two lower plates merely increase the freezing capacity. These plates are made to fit snugly into the freezing compartment and are placed about 5 in. apart. Between the shelf plates a double electric socket is fastened to each side of the cabinet. This permits placing four pans on each shelf and the strip heaters attached to each pan are plugged into these outlets to supply heat for thawing the specimens.

Watertight copper pans (3¼ x 15¾ x 15¼ in.) accommodate 16 specimens, and each pan is equipped with a 220-volt, 375-watt strip heater and has ⅛-in. rods soldered across the bottom to permit water to completely surround the specimens.

Concrete specimens are cast in 3 x 3½ x 15-in. steel molds, removed from the molds after 24 hours, and cured in the moist room at 73 F for 14 or 28 days. The specimens are then soaked in water and after 24 hours are transferred to the copper pans which are filled with water so that the specimens are completely submerged. The pans are placed in the freezing and thawing cabinet and the Flexopulse is adjusted to give a freezing period of 80 minutes which reduces the temperature of the exterior of the specimen to approximately −10 F and the center to + 5 F at the end of the freezing period. The thawing period is 40 minutes raising the temperature of the exterior of the specimen to 75 F and the center to 60 F at the end of the cycle. The freezing and thawing is continued until the average dynamic modulus of elasticity of the specimens in each group reaches 70 percent of the dynamic modulus of elasticity at zero cycles or through 200 cycles of freezing and thawing, whichever occurs first.

This equipment provides a rather severe freezing and thawing test. Most plain concretes fail in 25 to 50 cycles and air-entraining concretes resist 125 to 200 cycles.

Discussion of a paper by E. W. Scripture, Jr., S. W. Benedict and F. J. Litwinowicz:

Air Entrainment and Resistance to Freezing and Thawing*

By D. G. KRETSINGER and AUTHORS

By D. G. KRETSINGER†

The authors show some interesting results in their comparison of a fast and slow cycle of freezing and thawing. Even though a high early strength cement was used in the second of the two series made for comparison of cycle it is remarkable that in the slow cycle the non-air-entraining concrete lost only about 1 percent of its initial modulus as shown in Fig. 3 while the air-entraining concrete shown in Fig. 2 lost about 23 percent in 200 cycles. In connection with Fig. 2, the authors will probably want to correct the ordinate scale which indicates 10 percent more loss than actually occurred.

Differences in results between the fast and slow cycles used by the authors can probably be explained by Powers' hydraulic pressure hypothesis.‡ According to Powers, as freezing of the water progresses from the surface of a specimen inward, the unfrozen water in the saturated region of the surface will be displaced inward. Intensity of the hydraulic pressure developed during freezing depends on the rate of movement of the water which, in turn, is dependent on the rate of freezing. In these tests where freezing is so much more rapid in the fast cycle, deterioration should occur much more rapidly.

Some time ago the Bureau of Reclamation laboratories tested five air-entraining agents. These were: (A) a petroleum hydrocarbon insoluble fraction of a coal-tar hydrocarbon extract of rosin, (B) a triethanolamine salt of petroleum sulfonated hydrocarbon, (C) a neutralized solution of Agent A, (D) a detergent which is a neutral soluble sodium salt of processed rosin or resins derived from rosins, and (E) a tall oil which is a by-product of the sulfite Kraft paper process. These five agents were used to entrain approximately 4 percent air in 0.45 and 0.65 water-cement ratio concrete.

Freezing and thawing results of the concretes containing these five agents and the two water-cement ratios are shown in Table A. According to these results the five agents in the 0.45 water-cement ratio would be rated in order A,D,C,B,E, and in the 0.65 water-cement ratio concrete the rating would be B,A,C,E,D. This indicates that different agents will vary results of freezing and thawing durability tests and that this variation is subject to change when the agents are used to entrain air in concretes of different quality.

*ACI JOURNAL, Dec. 1951, *Proc.* V. 48, p. 297. Disc. 48-20 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 24, No. 4, Dec. 1952, Part 2, *Proceedings* V. 48.

†Bureau of Reclamation, Denver, Colo.

‡Powers, T. C., "A Working Hypothesis for Further Studies of Frost Resistance of Concrete," ACI JOURNAL, Feb. 1945, *Proc.* V. 41, pp. 245-272.

TABLE A—FREEZING AND THAWING RESULTS OF AIR-ENTRAINING CONCRETE

Agent used	Cycles of freezing and thawing to 25 percent weight loss			
	Water-cement ratio = 0.45		Water-cement ratio = 0.65	
	Percent air	Cycles	Percent air	Cycles
A	4.1	1140	3.8	570
B	4.2	720	4.5	620
C	4.1	810	3.9	430
D	3.9	900	4.2	280
E	4.2	570	4.0	350

Agents A and B are most used both in our laboratory work and in field concrete where entrained air is used. Other tests that we have made indicate there is little difference in freezing and thawing tests of concretes containing these two agents when air contents are the same. The entrainment of air is primarily responsible for increased resistance to freezing and thawing and we are in general agreement with the authors that the amount of air entrained is a more important factor than the agent used to entrain the air. However, Table A indicates that the agents used will produce variations in concrete durability even at the same air content. More caution is therefore needed in excluding the agents as a cause in variation of durability results.

We have made a series of tests to determine the effect of increasingly larger amounts of entrained air on freezing and thawing durability. Only two of these series, one for a 0.45 and one for a 0.55 water-cement ratio, are shown in Fig. A. All the tests indicate an optimum air content at which a maximum freezing and thawing durability is obtained. This optimum air content varies according to the quality of concrete, the better quality giving maximum durability at lower air contents. Although the better quality of concrete shown in this figure is obtained by variation of water-cement ratio, quality will also vary according to the aggregate or cement used, and therefore the optimum air contents shown are for the particular water-cement ratios, aggregate and cement used. The broken line curves labeled 4½ and 6 sacks per cu yd are from data in Tables 6 and 7 and Fig. 4 of the original paper.

It is apparent that had the authors carried their tests to higher air contents they would undoubtedly have shown an optimum air content for maximum durability. The trend of lower optimum air content for better quality concrete is not evident in their data. The reason is not readily apparent but may be due to the range of water-cement ratios (0.42 to 0.78) used.

Captions of Fig. 2 and 3 refer to loss of strength while the ordinates represent loss of dynamic modulus of elasticity. Modulus of elasticity is a measure of stiffness and as such does not always vary directly with strength.

AUTHORS' CLOSURE

There is undoubtedly an error in Fig. 2, in that the scale should be moved down one space; the corrected diagram appears with this closure. Mr. Kretsinger also calls attention to the captions for Fig. 2 and 3, which refer to loss in strength rather than loss of dynamic modulus of elasticity. Throughout the paper reference is made to losses in dynamic modulus and not to loss in strength. While the terms are not interchangeable, it has been fairly common practice to use them in this manner.

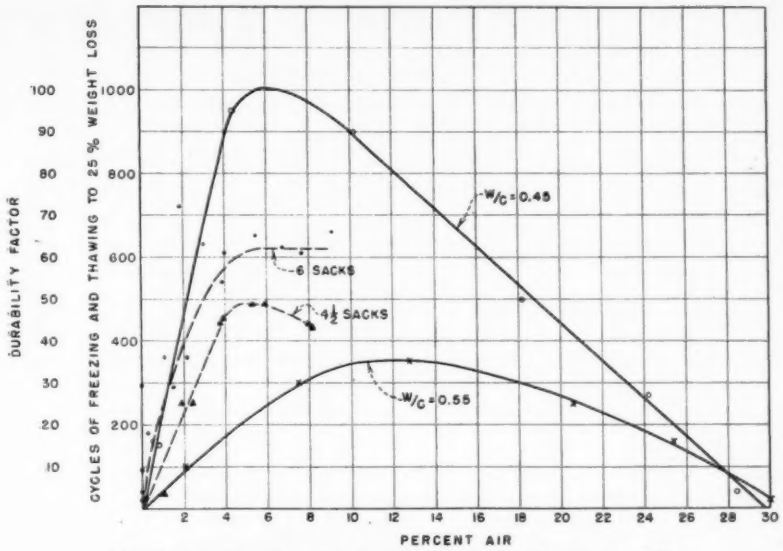

Fig. A—Relationship of air content to freezing and thawing durability

Results with the high early strength cement, which lost only about 1 percent of its initial modulus in 200 slow cycles, *are* rather surprising. This test series was run because a similar phenomenon had been observed by others with this particular cement. In those tests, involving 475 cycles under natural exposure conditions, with a 6 sack per cu yd concrete having a slump of 3½ in., the durability factor for this cement in a non-air-entraining mix was given as 103. A Type I cement made in the same plant, in this same series, had a durability factor of 27. A comparison between the results in Fig. 3 and those in Fig. 2 hardly seems to be justified since the cements, the cement factors, and the strengths in the two series were quite different.

The results which Mr. Kretsinger gives in connection with air-entraining agents A, B, C, D and E are quite surprising. From the description of these agents, C appears to be the same air-entraining agent as A except that it has been neutralized with sodium hydroxide. In view of the published information on air-entraining agents of this type it seems surprising that A, the un-neutralized agent, is preferred to C or that A gives better resistance to freezing and thawing than C. It would seem that A and C, provided that A is completely saponified in the mixer, should give the same results, unless there were an alkali-aggregate reaction involved which might have been promoted by the alkali in C. The description of D is somewhat vague but it would be expected that this material would be identical with C and would give the same results. Air-entraining agent B is a well-known agent, and if the authors' information is correct E is simply B without the triethanolamine

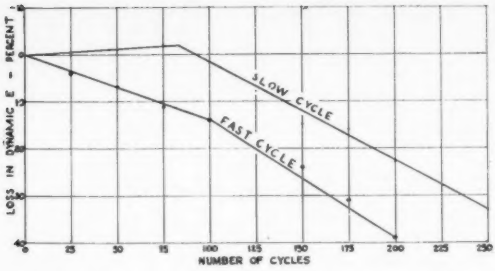

Fig. 2 (corrected)—Fast freezing and thawing increases loss of strength rate, Series I, Admix. A

component. Triethanolamine is not an air-entraining agent but is an accelerator. This introduces an extraneous variable into the picture, and it certainly is not the contention of the authors that the resistance to freezing and thawing secured with entrained air cannot be modified, for better or for worse, by accelerators, retarders, or various other additions to the mix.

It seems quite amazing that air-entraining agent B should give the next to poorest resistance to freezing and thawing in the mixes with water-cement ratio 0.45 and the best resistance to freezing and thawing in the mixes with water-cement ratio 0.65, while air-entraining agent D, which is, as far as is known, B without triethanolamine, gives the lowest resistance to freezing and thawing in the 0.65 water-cement ratio mixes and the next to highest in the water-cement ratio 0.45 mixes. This raises some doubt whether this particular group of freezing and thawing tests has really placed the air-entraining agents in order of merit with respect to their effects on durability. It seems as though there were some other factor or factors which influenced the results. The authors question the reliability of weight loss as a measure of durability. Cases have been encountered where specimens have shown no appreciable loss in weight or even a gain in weight and yet have lost over 50 percent in dynamic modulus and strength.

Mr. Kretsinger cautions against excluding air-entraining agents as a cause of variation of durability results. In this the authors agree entirely as stated in the paper, and conclusion No. 1 is qualified in this manner.

That there is an optimum air content for durability is not doubted, although the results which the authors have secured indicate that this optimum extends over a substantial range rather than reaching a sharp peak. It is well known that excessive air contents will seriously impair the strength of concrete, and it is to be expected that with lower strengths resistance to freezing and thawing will decrease more or less rapidly. For these reasons the authors did not pursue the investigation to air contents exceeding about 9 percent. Probably the third conclusion should have been qualified to state that the resistance to freezing and thawing increases to a maximum and thereafter remains substantially constant, provided that significant losses in strength are not produced by further entrainment of air.

Title No. 48-21

Streamlined Vacuum Concrete Buntons for Mine Shafts*

By PETER J. DOANIDES†

SYNOPSIS

Vacuum processed reinforced concrete beams have proved superior to steel and timber for structural bracing elements (buntons) in mine shafts in the gold fields of the Orange Free State, South Africa. Concrete buntons fulfill strength and impact resistance requirements with the additional advantage that they can be streamlined to offer less resistance to flow of air for ventilation and better resist corrosion.

Impact, bending and deflection tests are described. These tests of prototype buntons indicated certain weaknesses which were taken into account in redesigning the buntons.

INTRODUCTION

Vacuum processed reinforced concrete has found a new use in the gold fields of the Orange Free State in South Africa. Structural bracing elements (Fig. 1) in mine shafts, sometimes a mile or more deep, besides meeting strength requirements must resist corrosion, impact from ore spillage, and present minimum resistance to the free flow of air to reduce ventilation costs. Air currents set up by elevators traveling about 3000 ft per minute also increase power costs as much as several thousand dollars per year for each mine if buntons (horizontal struts) are not streamlined. Resistance of buntons to free flow of air inside a shaft greatly increases the power required for ventilation, and it is advisable to streamline them to reduce friction of the circulating air to a minimum.‡

Buntons have generally been built of timber or steel, but a new approach was deemed advisable because of prevailing corrosive conditions, with high humidity and temperatures requiring active ventilation of the mines. The corrosion is such that steel girders are seriously affected within a short time.

Reinforced concrete will outlast both timber and steel in corrosive surroundings and has the additional advantage that it can be precast in streamlined shapes and sections.

Sponsored by Anglo-Transvaal Consolidated Investment Company, Ltd., Johannesburg, tests on prototype buntons cast by the Concrete Develop-

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‡*De Ingenieur*, V. 62, No. 5, Feb. 3, 1950.

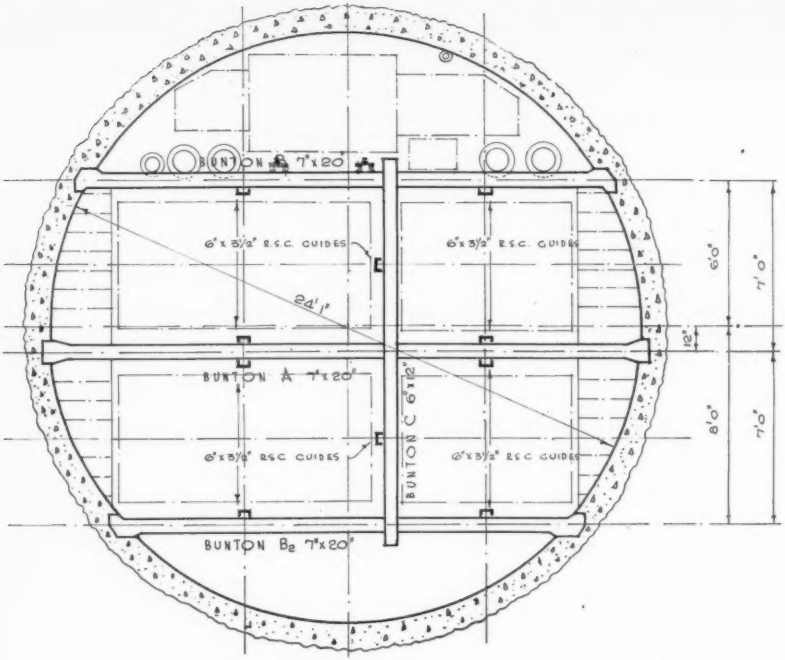

Fig. 1—Arrangement of concrete buntons for 24 ft 1 in. circular shaft

ment Corp., Ltd., and Roberts Construction Co., Ltd., proved that concrete buntons would fulfill all requirements.

To economically obtain first-class concrete, vacuum processing was utilized, achieving high strength in conjunction with early stripping of forms. The buntons were reinforced with longitudinal reinforcement and stirrups, plus interlocking spiral reinforcement to make them impact resistant.

DESIGN OF CONCRETE BUNTONS

The usual practice for circular shafts, in this case 24 ft 1 in. diameter, has been to use sets of steel buntons, each set comprised of three I-beams and one channel beam. (Similar to plan of concrete buntons, Fig. 1.) As the dimensions of steel buntons and shapes are based on empirical rules resulting from practical experience, some specifications were developed for designing concrete buntons: (1) concrete buntons should be equal to or better than equivalent steel buntons both in horizontal and vertical resisting moments and in horizontal and vertical deflections, (2) width of bunton should not exeed 6 in., (3) weight should be kept as low as possible to reduce handling costs.

With vacuum processing and high-early strength cement, ultimate stresses

exceeding 6000 psi can be obtained in the concrete. Assuming a cube strength of 5000 psi at 28 days, and in accordance with ACI specifications, a working stress of 1800 psi was accepted. As working stress for reinforcing steel, 20,000 psi was accepted instead of the 18,000 psi used for steel buntons because the reinforcing in concrete buntons, being protected by concrete, is not subject to corrosion.

The buntons should also resist impact from ore spillage down the shaft. Earlier tests showed that the resistance of concrete buntons is greatly improved by the use of interlocking spiral reinforcement.*

The exact amount and shape of such reinforcing must be arrived at by tests rather than calculations. After consultation with the South African Council for Scientific and Industrial Research, it was agreed to assume that usual spillage down a shaft consists of 4- to 5-lb. rocks. The velocity such a piece of rock can attain before it strikes a bunton exceeds 100 fps.

PROTOTYPE BUNTONS

Eight prototype buntons, 19 in. deep and 24 ft long, with 6-in. flanges, were cast following the designs shown in Fig. 2. Two types of spiral reinforcing were used: (1) two interlocking spirals of No. 10 gage wire, 2½ in. diameter at 2½-in. pitch and (2) two interlocking spirals of 3/16-in. wire, 2½ in. diameter at 2-in. pitch. Two types of stirrup reinforcement were used. In one case the stirrups corresponded to that required only for shear. In the other case, stirrups were placed at closer pitch to see if they would increase resistance to side impact. Two types of tensile reinforcement included that required by theoretical computations, and a reduction of 20 percent (under-reinforced) to find out if economies in steel could be effected.

To speed up testing, all test buntons were cast with a 2 percent addition of calcium chloride so that they could be subjected to impact and bending tests before 28 days. Sieve analyses of fine and coarse aggregate are listed in Table 1. A mix of approximately 1:1½:3 was used and sieve analyses indicated that the best aggregate grading was 35 percent ¾-in. stone, 35 percent ½-in. stone, and 30 percent river sand. The mix design for test batches is shown in Table 2.

TABLE 1—SIEVE ANALYSIS OF FINE AND COARSE AGGREGATE

Coarse aggregate—quartzite

Sieve	Percent passing	Sieve	Percent passing
1½-in.	100.00	¾-in.	100.00
1-in.	96.00	½-in.	97.72
¾-in.	54.08	⅜-in.	58.10
½-in.	2.76	No. 4	2.30
⅜-in.	0.56	No. 8	0.30
No. 4	0.44	No. 16	0.30
No. 8	0.40	No. 30	0.20
No. 16	0.34	No. 100	0.10

Fine aggregate—river sand

Sieve	Percent passing
No. 4	97.5
No. 8	90.3
No. 16	58.7
No. 30	17.7
No. 50	1.5
No. 100	0.2

*Report on concrete bunton linings and buntons with impact reinforcement, Concrete Development Corp. Ltd., Mar. 1950. Kluge, Ralph W., "Impact Resistance of Reinforced Concrete Slabs," ACI JOURNAL, Apr. 1943, *Proc.* V. 39, p. 397.

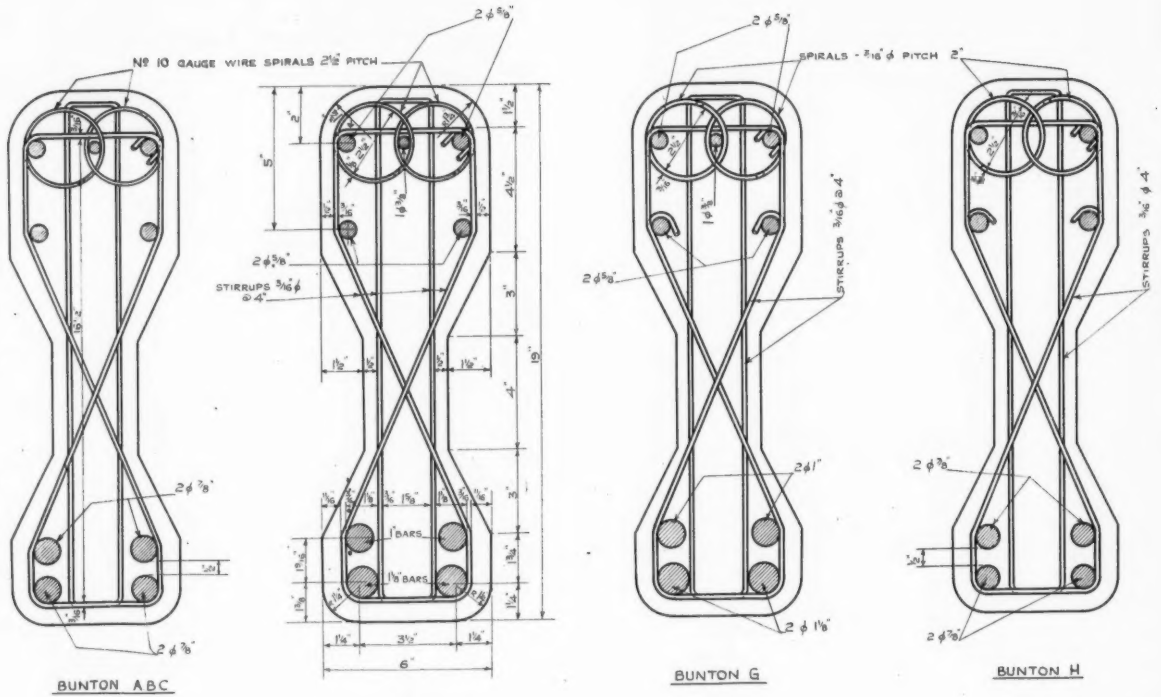

Fig. 2—Original design of sections of prototype test buntons

Tests were also made for compressive strength, according to American Standards Specifications. The test series included (1) plain concrete, (2) plain concrete vacuum processed for 20 minutes, and (3) vacuum processed concrete with the addition of 2 percent calcium chloride. The compressive strengths for the three series are compared in Fig. 3. The increase in strength for vacuum processed concrete is about 200 percent for one day and remains approximately 50 percent at 28 days. The calcium chloride admixture greatly improves early strengths, but the ultimate gain in strength is of no importance.

TABLE 2—CONCRETE MIX AND HANDLING

Cement	1 bag high-early cement (1 cu ft)
River sand	1¼ cu ft
Stone, ¾-in.	1½ cu ft
Stone, ½-in.	1½ cu ft
Water	50 lb
W/C	0.53
Slump	3 in.
Vibration	External
Curing	Kept damp for 7 days

Even though the number of test cylinders exceeded 100, the results should be considered as field rather than laboratory tests.

IMPACT TESTS

The South African Council for Scientific and Industrial Research tested the vacuum processed concrete buntons for impact resistance. The main object was to determine the damage a concrete bunton would suffer from rock spillage. A pointed missile, weighing the same as a large average piece

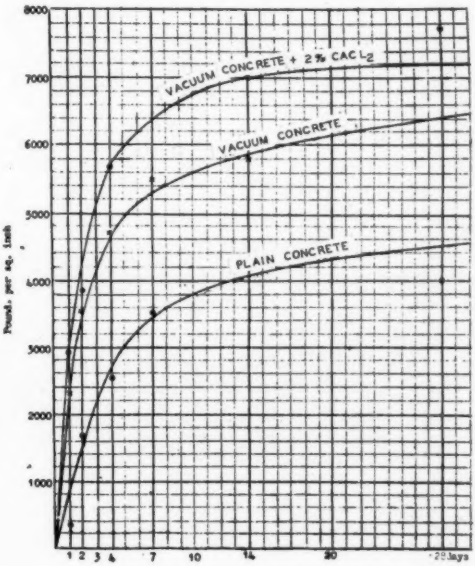

Fig. 3—Comparison of compressive strength of plain and vacuum concrete

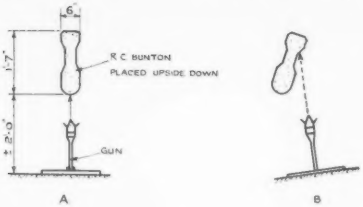

Fig. 4-A—Arrangement of bunton and grenade thrower for impact tests. B—Grenade thrower in position for firing at exposed flange of bunton

of spillage, was fired at the buntons at a speed corresponding to the maximum velocity that could be attained by a piece of rock falling down a shaft. To produce realistic average conditions two steel missiles, 3¾ and 2¼ lb, were used. They were cylindrical with a rounded conical point. Striking velocities were selected to correspond to the maximum velocity likely to be attained by freely falling rock in the shaft. For a spherical rock 2½ in. in diameter, the maximum velocity would be 213 fps after falling about 700 ft. For the average piece of irregularly shaped rock, the air resistance is increased so its highest velocity would be appreciably less.

Test method

Each bunton was supported on a trestle and placed in an inverted position so that the face of the bunton covering the impact reinforcement was about 2 ft. above ground (Fig. 4a). A grenade thrower was mounted on a flat steel plate with its barrel in a vertical position so that the missile was fired upward. A 32-grain ballistic charge was used to get as high a striking velocity as possible. The actual firing of the gun was by remote control. A few shots were also fired at the exposed flange of the bunton (Fig. 4b), which under rare circumstances could also be hit by falling ore. The average striking velocity of the 3¾-lb missile was 120 fps.

Results

The damage resulting from one shot of the 2¼-lb missile was about the same as for one shot of the 3¾-lb missile (Fig. 5). When several shots were

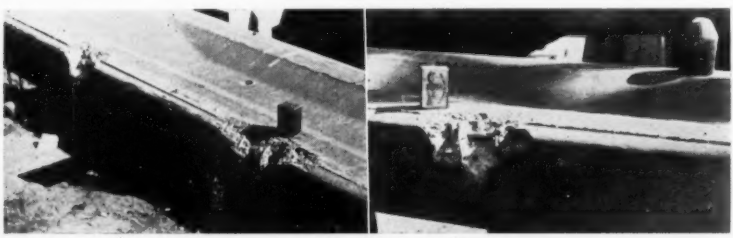

Fig. 5—The first seven shots were fired vertically upward at random to hit the face covering the impact reinforcement of bunton B. The 2¼-lb missile was used for shots 1 to 4 (left); the 3¾-lb one for all remaining shots. The degree of damage resulting from each of the seven shots was about the same, consisting of spalling off of the concrete over about 15 to 20 sq in., enough in some cases to expose part of the reinforcement.

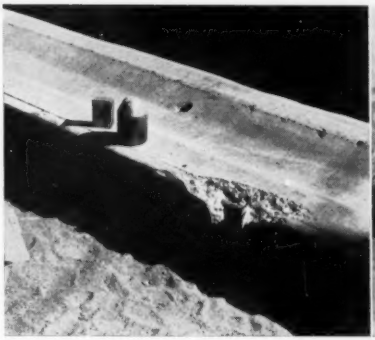

Fig. 6—Five shots, using the 3¾-lb missile, were fired at exactly the same place on bunton E. The effect of each shot was: (shot 1) spalling off one corner; (shot 2) spalling off other corner and exposing spiral as well as small piece of longitudinal reinforcement; (shot 3) no additional damage visible; (shot 4) spalled small amount off center of bunton and produced small surface crack; (shot 5) little additional damage.

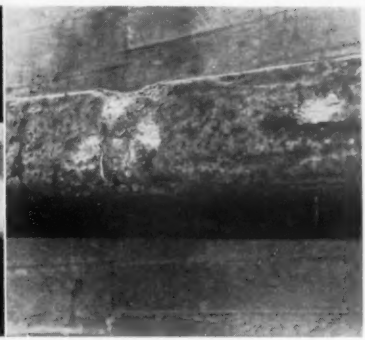

Fig. 7—Five shots, using the 3¾-lb missile, were fired at the same place on bunton H with the following results: (shot 1) slight spalling on one side; (shot 2) slight spalling on other side; (shot 3) additional damage negligible; (shot 4) no additional damage visible; (shot 5) slight additional damage to exposed part of reinforcement.

fired at one spot the first shot generally produced most damage with subsequent shots causing little additional damage (Fig. 6 and 7). The total damage was confined to spalling off of the concrete shell covering the reinforcement and in some cases exposing the spiral and longitudinal reinforcement. Damage sustained by the heavier impact reinforced buntons was less than that of the buntons with lighter impact reinforcing. Damage to the exposed flange was almost negligible (Fig. 8), due mainly to the fact that the missile was deflected off at an angle.

It is probable that five shots fired at the same spot, using a sharp-pointed 3¾-lb missile traveling at 120 fps produced a condition more severe than would be obtained from average spillage.

In general the impact tests gave favorable results for the vacuum processed concrete buntons. The spiral reinforcement proved effective and no rupture occurred, only spalling of the concrete surface. The tests showed that the concrete is protected from spalling by the spirals so it was recommended that the spiral impact reinforcement be placed as close to the surface as possible, allowing only minimum cover to protect the steel against corrosion. It was also recommended that the pitch of the spirals be about 1½ in. to reduce the possibility of penetration by falling objects to points deeper than the spirals.

Although the diameter of spiral reinforcement is not a deciding factor in improving impact resistance, it was considered advisable to use 3/16-in. wire instead of No. 10 gage to allow more material in case of exposure and danger of corrosion.

Fig. 8—When the missile was fired at an angle to hit the exposed upper flange of the bunton there was only slight surface damage

BENDING AND DEFLECTION TESTS

Tests for bending and deflection of the buntons were made in the Department of Civil Engineering of the University of the Witwatersrand, Johannesburg. The buntons were tested for load-deflection behavior and ultimate loads in two directions, on edge and side. Two series were run, one on buntons damaged by impact tests and one on the undamaged buntons.

Each bunton was supported on rollers (Fig. 9) to span 24 ft and a pair of equal loads was applied through knife-edge supports located 2 ft on each side of the mid-span point. Fig. 10 shows general arrangement for recording deflections. Six buntons were tested with the 19 in. dimension in a vertical direction; two were tested on their sides. Loading data appear in Table 3.

Results

The actual values of the multiple EI found from the bending tests compared favorably with the values used in design.

To compare the resisting moments of concrete buntons with steel girders

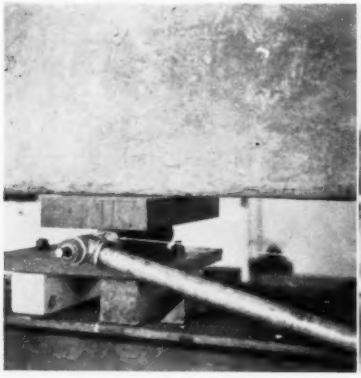

Fig. 9—Roller support at one end of bunton showing end of truss used for recording deflection.

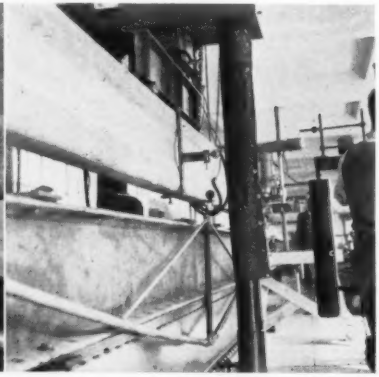

Fig. 10—General arrangement of loading and gear for deflection recording.

TABLE 3—BENDING AND DEFLECTION TESTS

Bunton	Test position	Damaged in impact test	Loading cycles	Failure lb	Remarks
A	Edge	No	4	26,410	At failure large tension cracks opened up over central portion of span. As deflection increased, concrete failed in compression near center of bunton.
B	Edge	Yes	1	25,490	At failure tension cracks opened up over central portion. As deflection increased, concrete failed in compression at point where compression surface had been damaged by impact test missile.
C	Side	No	1	3,680	At failure tension cracks opened up over central portion of span. Maximum deflection of approximately 6 in. It was not possible to produce sufficiently large deflections to cause failure in concrete.
D	Edge	No	5	30,930	At failure tension cracks opened up over central portion. As deflection increased, concrete failed in compression at one of knife-edge loads.
E	Edge	Yes	2	28,900	At failure tension cracks opened up over central portion and as deflection increased, compression failure occurred in an undamaged portion of concrete surface about 2 ft from a previously damaged place.
F	Side	No	1	4,560	At failure tension cracks opened up over central portion. Maximum deflection approximately 6 in. It was not possible to produce sufficiently large deflections to cause failure in concrete.
G	Edge	Yes	3	31,150	At failure tension cracks opened up over central portion. As deflection increased, concrete failed in compression at a point where previous testing had damaged the surface.
H	Edge	Yes	1	25,090	At failure tension cracks opened up over central portion. As deflection increased, concrete failed in compression at a point where previous testing had damaged the surface.

(Table 4), the following must be considered. All concrete buntons failed in tension steel. The design working stress of the reinforcing steel was f_s = 20,000 psi, or 10 percent higher than the working stress for the steel girder, allowing a 10 percent decrease in the safety coefficient because of efficient protection of the reinforcement against corrosion. In comparing the resisting moments before the assumed corrosion of the steel had come into effect, one should decrease the resisting moments of the steel girders by 10 percent. Naturally the under-reinforced concrete buntons were weaker than the steel girder.

The flexural rigidity of concrete and steel buntons are also compared in Table 4, where the flexural rigidity of the steel girder was reduced 10 percent to allow for weakening through corrosion.

REDESIGN OF BUNTONS

The information learned in the impact, deflection and bending tests made possible the redesigning of the concrete buntons to meet more fully the needs

TABLE 4—COMPARISON OF MOMENTS OF RESISTANCE AND FLEXURAL RIGIDITIES

Type of beam	Moment of resistance, in.-lb		Flexural rigidity, EI, psi $\times 10^6$	
	Edge	Side	Edge	Side
Rolled steel joist, 12 x 6-in., 44 lb, estimated	Min. 1,835,000 Ave. 2,076,000 Max. 2,404,000	256,200 289,900 335,700	8,554	597
Concrete buntons with design reinforcement				
Buntons D,E,G	1,900,000		10,543	
Bunton F		353,000		637
Concrete buntons under-reinforced				
Buntons A,B,H	1,618,000		8,467	
Bunton C		299,000		523

TABLE 5—COMPARATIVE STRENGTH DATA FOR STEEL AND CONCRETE BUNTONS OF REVISED FINAL DESIGN

Buntons	Characteristics	Calculated resisting moment, in.-lb		Calculated flexural rigidity EI, psi $\times 10^6$	
		Vertical bending	Horizontal bending	Vertical bending	Horizontal bending
A*	Steel, RSJ, 12 x 6-in. x 44 lb	950,220	132,660	9504	663
	Concrete, 7 x 20 in. A_s = 2x1-in. ϕ + 2x1⅛-in. ϕ	1,072,000	220,800	15,310	1412
B*	Steel, RSJ, 10 x 6 in. x 40 lb	737,280	130,500	6144	653
	Concrete, 7 x 20 in. A_s = 2x⅞-in. ϕ + 2 x 1-in. ϕ	748,000	186,400	14,127	1382
C*	Steel, RSC, 10 x 3½ in. x 24 lb	394,000	52,700	3285	222
	Concrete, 6 x 12 in. A_s = 2 x 1-in. ϕ	258,488	56,900	2262	325

*See Fig. 1 and 11.

for mine shafts (Fig. 11). The beams were redesigned to use 7-in. flanges instead of 6 in. Three spirals, 3/16-in. diameter, 1½-in. pitch, were used with the originally designed reinforcement. Bunton C, acting merely as a stiffener (Fig. 1) in case of horizontal deflections, was shaped accordingly without taking into consideration comparative data with the steel girders it replaces. Table 5 compares the new design with the steel girders formerly used in the mine shafts.

ACKNOWLEDGMENTS

Valuable cooperation and assistance was received from A. C. Backeberg and C. Skeen of Anglo-Transvaal Consolidated Investment Co., Ltd., J. E. Jennings, N. Stutterheim and A. K. Steyn, South African Council for Scientific and Industrial Research, and A. J. Ockleston, R. C. Ventrees and W. H. King, Civil Engineering Department, University of the Witwatersrand, Johannesburg.

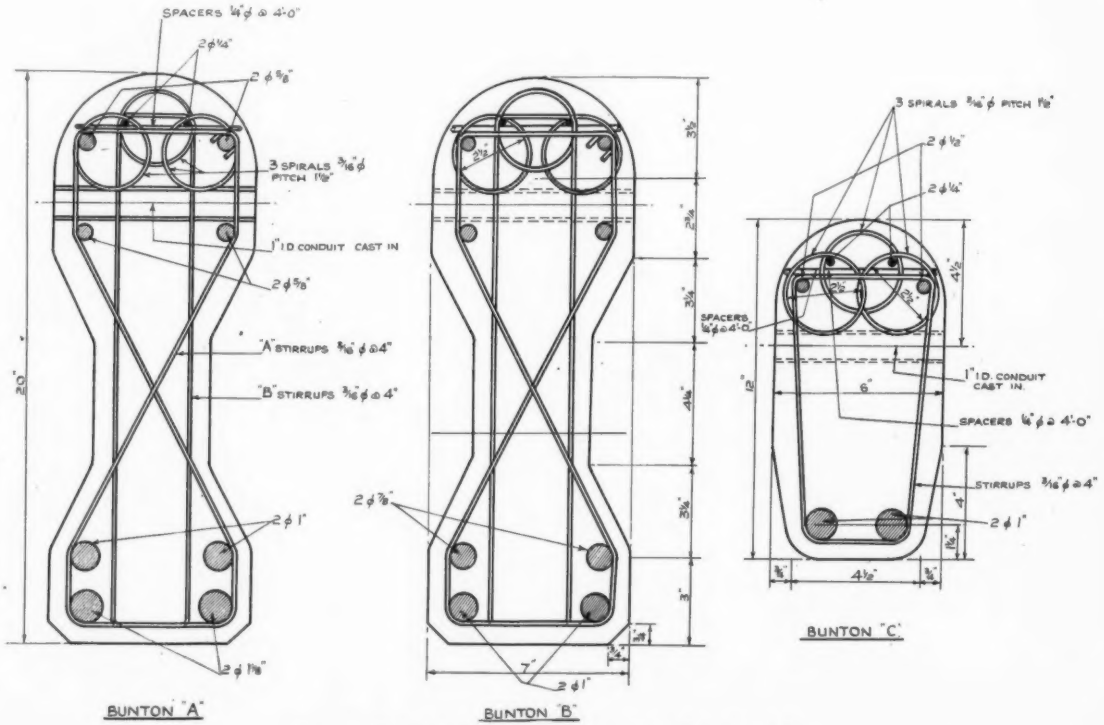

Fig. 11—Redesigned sections of buntons as used in mine shafts

Title No. 48-22

Simple Concrete Shell Structures*

By FELIX CANDELA†

SYNOPSIS

Some small simply-designed reinforced concrete shells constructed in Mexico City are described. Included are three types of cylindrical shells and two skewed shell structures. They were erected by relatively unskilled labor at a competitively low cost and provided novel solutions to owners' requirements.

INTRODUCTION

Most technical papers concerning design of reinforced concrete shells emphasize the difficulties of their analytical calculation, a fact which favors the general opinion that these structures are suitable only for daring long-span roofs, because in other cases the extensive numerical computations deemed necessary are not economically justifiable. The result is that cost of expensive calculations must be added to an exorbitant cost of formwork required for such gigantic structures, making their effective use impracticable except when cost is not the decisive factor. Nevertheless, reinforced concrete shells are, if designed upon a reasonable basis and for moderate spans, more economical than the usual sheltering solutions.

In view of the impossibility, when dealing with a structural problem, of taking into account all the secondary conditions, such as contractions due to the setting of concrete, temperature differences between points in the structure and uneven settlement of foundations and considering the intrinsic roughness of construction materials and inevitable lack of precision in building it is obvious that mathematical calculation can never afford an exact and unique solution. Hence the inutility of complex design methods. It is better to use simpler procedures which, in most cases, are sufficient when the designer is a constructor and not merely a computer.

The imposing stone vaults of Gothic cathedrals and the daring domes of the Renaissance were built without help of differential calculus but, instead, with a great deal of sense of equilibrium and sound judgment of the play of forces, qualities more necessary indeed to a real builder than full knowledge of mathematical intricacies.

There is no fundamental difference, apart from thickness, between stone or brick vaults and domes and reinforced concrete short cylinders or spherical

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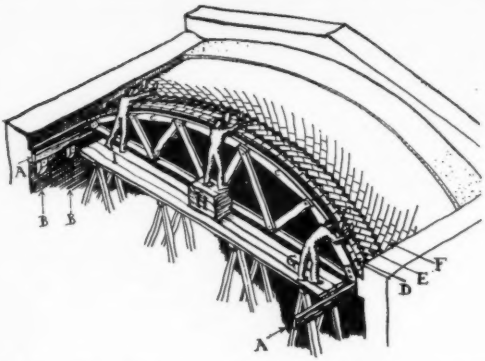


Fig. 1—Building a brick vault composed of three layers of thin hollow brick (From *Bovedas Tabicadas*, by Luis Moya)

shells. Even thinness is not a special characteristic of the latter. It is habitual in Spain, following a Mediterranean tradition, to employ vaults and domes made from a particular type of 1-in. hollow brick (called "rasilla") forming structures usually little more than 3 in. thick[2] (Fig. 1). Sometimes this thickness is reduced to only one layer of 1¾ in. hollow brick.[3] After the last Civil War application of this traditional procedure substantially increased due to the scarcity of steel. The Spanish architect Guastavino built a number of such structures in the United States about 1900.

These brick shells require, as a general rule, only approximate calculations. The election of an appropriate surface form, rough confirmation of maximum normal stresses and disposition of stiffening brick arches at distances dictated by building tradition, are usually the only requisites. As a matter of fact, even the drawing of the more complicated shapes, such as the helicoidal ramps of staircases, is left to skilled masons on the job.

Reinforced concrete has the advantage over such brick structures of being capable to admit tensile and flexural stresses and does not require skilled workers. Therefore it seems difficult to find reasonable objection to wide use of reinforced concrete shell structures of moderate size as logical solutions to the sort of construction and architectural problems for which they are specifically indicated.

SHORT SHELLS

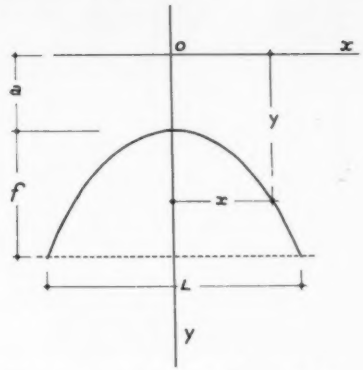
The simplest form of concrete shell is the cylindrical vault of antifunicular cross section, bearing at the springings. In this climate (Mexico City) it is not necessary to consider snow loads. For flat shells, wind loads are only suctions and can be considered as evenly distributed upon the surface, reducing the working stresses.

Therefore the funicular line corresponds, almost exactly, to dead load only and for a shell of uniform thickness becomes a catenary. The analytical expression of such a curve is the well-known formula

$$y = a \cosh (x/a)$$

in which the parameter a depends on f and L (Fig. 2).

Fig. 2—Diagram of catenary



The vertical load per unit surface being p, the horizontal thrust is $H = p \times a$ per unit length. The maximum compressive force at the springings is $R = p(a+f)$, per unit length. The shell at the springings acts as a deep beam transmitting these forces to the points of application of the tie rods and supporting points.

Cylindrical shell

In a bowling alley designed by José Domingo Samperio, the tie-rods are placed above the shell and the heads of the supports are projected upward and, as a result, must resist flexural moments in addition to vertical loads, a fact which justifies their unusual form (Fig. 3). Apart from esthetic reasons this disposition of the tie-rods makes traveling formwork easy.

For moderate spans, the thickness of the shells is determined only by practical reasons, the stresses being certainly very small, and the general reinforcement is purely nominal. A mesh of 3/16- or ¼-in. round steel 8 in. on

Fig. 3—Roof showing projecting supports, tie rods and stiffening ribs of concrete shell

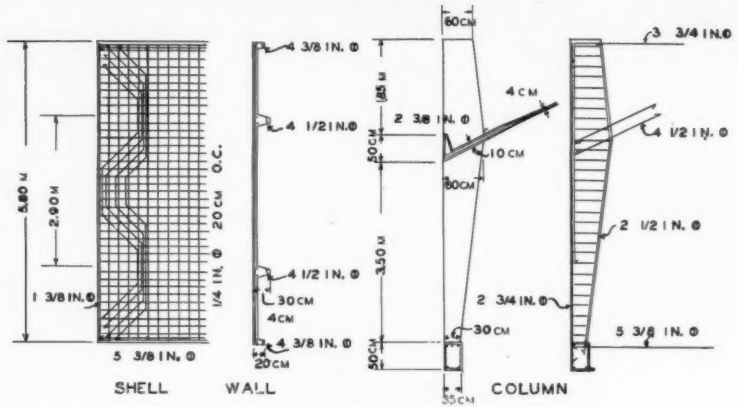


Fig. 4—Reinforcing details of structure shown in Fig. 3

centers sufficing for temperature purposes. Additional reinforcement, suitable to the beam action of this part of the shell, must be arranged at the springings (Fig. 4). It is also convenient to provide at the same places two layers of ordinary transverse reinforcement to take care of transverse bending moments resulting from edge disturbances.

In this case the general thickness is 4 cm (1½ in.), increasing to 4 in. at the springings in a width of 4 ft. The span is 15 m (50 ft), with a rise of about 6 ft. The longitudinal distance between supports is 10 ft. The total length of 20 ft of each shell element being completed by two 5 ft. (Fig. 5) cantilevers.

The maximum working normal stress is 92 psi (about 7 kg per sq cm). The danger of buckling is obviated by stiffening ribs of small cross section. Similar but smaller ribs are disposed at the ends of the overhangs, both to strengthen these weak zones and to protect construction joints.

For computing safety against buckling the following empirical formula, developed by H. Lundgren[4] after a series of tests can be applied:

$$\text{Critical stress} = \sigma_{RK} = \frac{\sigma_T}{1 + \frac{2840 + \sigma_T}{7{,}100{,}000} \cdot \frac{l}{d}\sqrt{\frac{R}{d}}}$$

Where

σ_T = rupture stress of concrete, psi
l = Distance between stiffening ribs, in.
d = thickness of the shell, in.
R = Radius of curvature of cross section, in.

In this case the critical stress is

$$\sigma_{RK} = \frac{2000}{1 + \frac{4840}{7{,}100{,}000} \cdot \frac{120}{1.5}\sqrt{\frac{540}{1.5}}} = 1020 \text{ psi}$$

which represents a factor of safety of eleven.

Fig. 5—Interior of bowling alley after removal of arch forms

The first step in construction was concreting the low part of the columns of each structural element, together with the springings of the shell and the edge beams forming the gutter (Fig. 6). As the whole structure is composed of five independent units, the formwork could be reused ten times.

The second stage was to pour the heads of the columns, employing a mold reused 20 times. The ends of the tie rods were left projecting from these heads to be completed later by addition of a central part, attached and put in tension by an inexpensive device (Fig. 3).

The last operation, concreting the central part of the shells including the stiffening ribs, was done in five steps with the help of a very simple traveling form mounted on four small rubber-tired wheels. Four 2-ton hydraulic jacks were used to level and strike this centering. The concreting of the stiffening arches was made without help of lateral forms. Striking the centering, translating it to the next pouring position and leveling took only two hours, and the total interval between two consecutive concreting operations

Fig. 6—Concreting of columns (left foreground) and preparation for placing first shell section

Fig. 7—Formwork and reinforcing for reinforced concrete vaults. Completed section in background

was a week. 1:2:4 rapid hardening concrete was used for the whole work, the maximum size of coarse aggregate for the shell being ⅜ in.

Series of cylindrical vaults

In another building (Fig. 7), a sporting-goods factory designed by Raúl Fernández R., the thrust of a series of cylindrical vaults is taken by oblique compression members at the end of the row. The almost square ground floor (8 x 10 m or about 27 x 33 ft) of each vault (shells of intermediate length) makes rather unorthodox the assumption adopted for the carrying action of the junction of two contiguous shells. The horizontal thrust of each vault being counter-checked by the adjacent ones, the resultant at the joint is a vertical force which is taken by the angular member formed by the two springings. In fact, such a member is able to resist an applied force of arbitrary direction and its behavior in practice has been absolutely satisfactory.

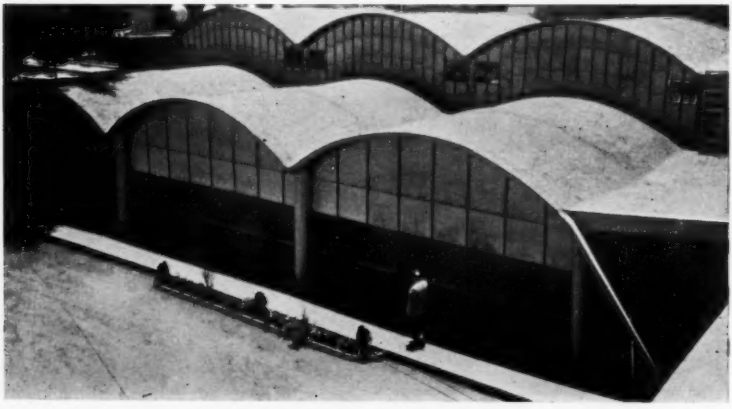


Fig. 8—Building in Fig. 7 after completion

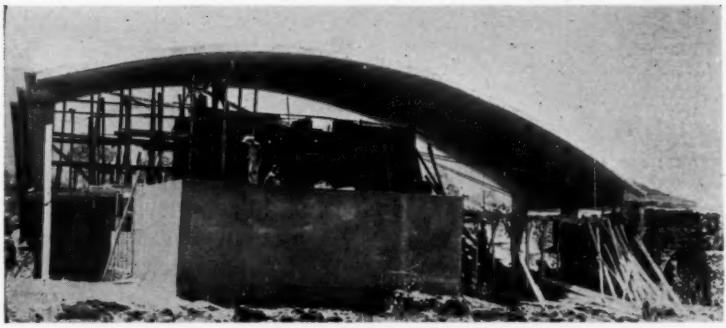


Fig. 9—Completed shell roof for "Novedades" house

The dimensions of each vault are: span, 8 m (27 ft); longitudinal distance between supports 10 m (33 ft); thickness of the shell 3.5 cm (1⅜ in.) increasing to 4 in. at the springings. Stiffening ribs were replaced for esthetic reasons by an increase in shell thickness to 4 in. in three strips 1½ ft wide at the two ends and center of each vault.

The whole structure is composed of nine vaults in three rows, of which the intermediate is at a higher level to allow for two strips of windows, which results in pleasant and satisfactory illumination (Fig. 8).

It is interesting to note that the bidding price of these structures was lower than competitive solutions with steel trusses and corrugated asbestos sheets, being always less than one dollar per sq ft, including concrete pavement slabs.

Unsymmetrical funicular vault

Fig. 9 and 10 illustrate one of the first cases, or perhaps the only one, in which reinforced concrete shell technique has been applied to residential building work. It shows a house, designed by Raul Fernandez R. and the

Fig. 10—"Novedades" house after completion

Fig. 11—Conoidal reinforced concrete shell

writer, and built in the newest and most exclusive residential district of Mexico City (Pedregal de San Angel).

The reinforced concrete shell covering the whole house is an unsymmetrical funicular vault, spanning 19 m (63 ft). The longitudinal distance between stiffening arches and supports is 5 m (17 ft), the general thickness is 5 cm (2 in.) and the reinforcement and detailing are similar to the previous examples. The tie rods are obliquely disposed under the back arch and form broken lines under the forward ones to comply with the architectural requirements.

SKEW SURFACES

Skew surfaces present very interesting properties from the point of view of shell construction. Their double curvature makes them exceptionally resistant to buckling, allowing a reduction in required thickness to a minimum and, in some cases, even the elimination of stiffening ribs. As they are generated by straight lines, this property can be very useful in the design and erection of forms. The more usual types are the hyperboloid, the hyperbolic paraboloid and the conoid.

Conoidal shell

Fig. 11 represents a conoidal shell, very suitable for saw-tooth roofs, designed following the same simple principles already mentioned. The spans are 15 x 6 m (50 x 20 ft), the thickness 3 cm (1¼ in.) increasing to 4 in. at the springings and the cross section of the stiffening ribs is 10 x 20 cm (4 x 8 in.). The tie rods are above the shell, as in the first example, to expedite moving the form work.

Coupled hyperbolic paraboloids

Fig. 12 shows an unusual structure. Designed in collaboration with architects Gonzalez Reyna y Arozarena to lodge the Cosmic Rays Laboratory of "Ciudad-Universitaria" in Mexico City, it was a functional requirement that the top part of the shell should have no more mass than 40 kg per sq m (about 8 psf). Therefore the thickness had to be reduced to 1.5 cm (⅝ in.)

Fig. 12—Shell for cosmic ray pavilion after concreting and removal of centering

in the upper half of the vault, resulting in perhaps the thinnest permanent reinforced concrete structure of its size ever built.

In view of the required thinness additional precautions were taken against buckling, adopting the form of two hyperbolic paraboloids coupled along a principal parabola. Such skew or nondevelopable surfaces can be considered as generated by two systems of straight lines, each system being parallel to a vertical plane. In this case, the two director planes form an angle of 60°. Each generatrix of a system intersects all the generating lines of the other one, but does not touch those of its own system, being in parallel planes with them.

Profiting by this property of the surface, the forms were ½-in. tongue and groove flooring board laid, in an approximate form, according to a generating system, on 2 x 4-in. wooden joists disposed exactly along the other system of straight generatrix (Fig. 13). The reinforcement, disposed along the principal parabolas, was a mesh of ⅛-in. mild steel wire placed 4 in. on centers.

The span of the vaults is 10 m (33 ft), the distance between the three stiffening arches, whose cross section is 10 x 12 in, is 5 m (16½ ft), with two overhangs of 90 cm (3 ft). The 1.5 cm (⅝ in) thickness increases to 2 in. toward the springings. The cantilevers are 4 cm (1½ in.) thick.

The comparatively high rigidity of the beams upon which the vault bears, nullifies practically all edge disturbances, resulting in a shell free from bending stresses.

The precautions taken against buckling and the general disposition of the shell made any accurate calculation scarcely justifiable. A rough confirma-

Fig. 13—Formwork for cosmic ray pavilion

tion can be made, determining the maximum normal stresses in a cylindrical funicular vault of the same span and rise, whose cross section differs but very little from the real one. This artifice gives an idea of the magnitude of stresses in the actual shell, about 2 kg per sq cm (30 psi). For more critical cases, the exhaustive work of Aymond[5] on such skew shells, can be consulted if necessary. The application of the abacus[5] confirms the insignificance of the real stresses.

Neglecting, in the same way, the double curvature of the surface, the previously mentioned Lundgren's formula[4] gives a factor of safety against buckling of about 9.

Considering all this, it can be said that this was only a mason's or carpenter's problem, the chief difficulties being the establishment of the centering and the pouring operation. This last job was executed by hand, as in the previous examples, in 8 hours, without complications in spite of the thinness and exaggerated slope of the shell. 1:3:2 were the proportions of the mix, with ¼-in. maximum size coarse aggregate. The forms were removed at the fourth day, as usual with rapid-hardening cement.

No measurement of the deformations of the shells was planned, but its behavior seems satisfactory and it resists concentrated loads of moderate size without apparent deformations.

SUMMARY

The small structures described demonstrate that shell construction is not limited by design requirements or comparative cost to exceptionally long

spans. Following simple designs, the buildings were erected economically and quickly without special equipment. Those described were built by Cubiertas Ala, S. A., Mexico D. F.

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Discussion of a paper by Felix Candela:

Simple Concrete Shell Structures*

By ERIC C. MOLKE and AUTHOR

By ERIC C. MOLKE†

Mr. Candela's appeal for simplified and practical shell design is timely and most refreshing. As he points out, the use of more empirical design considerations, instead of involved and expensive calculations, opens up a new field in the application to smaller structures.

Several favorable conditions seem however to exist abroad and particularly in Mexico, the lack of which in the United States hampers wider application of such efficient structural elements.

"Shell" action differs from conventional two-dimensional arching mainly in the way in which the thrust is channelled along the shell surface to the stress concentrations at the column supports. With the comparatively "short" shells illustrated by Mr. Candela, this involves merely a bridging of the arch thrust between columns, almost similar to the well-known "deep girder" action described in Portland Cement Assn. *Bulletin* No. ST 66. The only difference comes from the fact that with a shell the deep girder is not a plane but a curved surface, which causes a transverse or "slab" bending action near the springing line. Thus shell action can be reduced to a combination of conventional arch, girder and slab action, for which a rather rigidly defined design procedure is called for in our codes.

Concrete stresses being generally low with shell structures and buckling danger being particularly low with the small structures illustrated by Mr. Candela, his shells can be extremely thin. The governing limitation is merely the necessary fireproofing cover called for by code or convention. As in many other countries, this minimum cover seems to be 1 cm in Mexico, while our code calls for a minimum covering of ¾ in., or twice as much.

The live load requirements are also in favor of these structures in Mexico. A cylindrical vault shaped to the catenary is ideal for uniform loads, but if snow loads are specified, we must provide substantial stiffening ribs. It must be noted that the primary reason for stiffening ribs in our country is not buckling, but bending under one-sided live loads.

From the foregoing it can be concluded that for similar structures designed for the requirements in the United States, the dead loads would become at

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Fig. A—Small warehouse built with precast curved slab panels in the steeper portion of the shell

least twice as large, causing a proportional increase in reinforcing steel and column loads.

It seems further that the application of shells to smaller structures in this country will only be successful where a proper number of repetitions is called for, not only to save on the high cost of forming, but also to give sufficient incentive to overcome the common reluctance to bidding on any unconventional type of work. The writer therefore believes that precasting sectional shell panels and erecting them with cranes is the method most suitable for conditions in this country.

A step in this direction was the recent construction of a small warehouse of 80-ft span, with 29 ft rib spacing, designed and built by the Bailey Co. in Fort Worth, Texas, with the writer as consultant, where precast curved slab panels have been used successfully for the steeper shell portions (Fig. A).

Our codes are a little more lenient in regards to the minimum cover over reinforcing with precast work, where "positive and rigid devices" are used for locating the reinforcing. Precast panels can also be erected and tilted at an angle against each other, whereby we can easily obtain a "corrugation" effect, in principle similar to the action of skew surfaces as employed by Mr. Candela. In this way we can also obtain the extra bending stiffness for the shells, which we need to satisfy the requirements for one-sided snow loads, without having to resort to cumbersome and heavy stiffening ribs.

Thus a great impetus to the use of shell structures should come in the near future, particularly when the public learns to appreciate the extra resistance of shell structures to unforeseen catastrophic loads, such as those caused by fire and bombardment, advantages which can be obtained by using a minimum of critical materials.

AUTHOR'S CLOSURE

The author wishes to thank Mr. Molke for his comments and agrees with him that climatic and regulative pecularities which condition design are different in our two countries. Perhaps another way to overcome the fireproofing cover limitation would be to omit the reinforcement, at least at some parts of the shell. This procedure, which may appear drastic to many designers, is possible in many cases of short shells and especially when employed in connection with longitudinal corrugations, as suggested by Mr. Molke or as successfully used by Mr. Billig in his "corrugated concrete arches."

The author is also aware of the essential differences in labor and materials costs as compared with United States conditions, which seem at first to favor the employment of shell structures in Mexico. But the substantial repetition of identical elements and employment of traveling formwork is a condition as necessary in Mexico as in the United States to make the over-all cost comparable with competitive types of construction.

It seems, however, that the true reason for limited acceptance of shell roofs must be sought in the reluctance of owners to accept any change in the conventional type of roof, and chiefly in the general impression held by the average practicing engineer that the design of a shell is a highly scientific problem requiring specialized knowledge. The author's paper was directed against this erroneous opinion, intending to remove some of the mystery which, for reasons perhaps not purely scientific, has surrounded the design of shells.

Title No. 48-23

Concrete Footings for Walls and Columns*

By PAUL JAKOWLEW-HERBACZEWSKI†

SYNOPSIS

Equations are developed from which charts and tables are prepared which permit the determination of optimum dimensions for concrete footings. The effect of varying costs of materials is considered and a method is given for determining the economical percentage of steel in concrete.

INTRODUCTION

The aim of the equations, charts and tables which follow is not only to aid the designer in saving time and tedious labor of "cut and try" methods, but to present a clear and broad view of the whole range of possibilities in dimensions and proportions open to the judgment and choice of the designer in figuring wall or column footings. No new theory is presented, but from commonly known equations and relations graphs have been prepared representing the necessary data for a rapid selection of the best footing for the job at hand. The data and curves for diameters of bars as well as equations for economical percentage of reinforcement are perhaps novel. Their demonstrated usefulness recommend their wider application by designers.

NOTATION

P = Total load on column or on 1 ft length of wall, including column or wall weight, tons

Q = Surcharge load on footing outside of wall or column (not included in P): Live load, dead load, fill and own weight of footing, tons

$P + Q$ = Total load on bearing soil, tons

B = Width of wall footing, ft

B_1 and B_2 = Length and width of rectangular column footing, ft

C = Width of wall or column, ft

q' = Surcharge unit load on footing outside of wall or column

$$q' = \frac{Q}{B - C} \text{ tons per sq ft—for wall}$$

$$q' = \frac{Q}{B_1 B_2 - C^2} \text{ tons per sq ft—for column}$$

q = Actual soil unit pressure under footing base (equal to or less than the allowable soil pressure)

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$$q = \frac{P + Q}{B} \text{ tons per sq ft—for wall}$$

$$q = \frac{P + Q}{B_1 B_2} \text{ tons per sq ft—for columns}$$

w_t = ton per sq ft = $q - q'$ = Net pressure producing bending in footing
L = clear span of projecting part of footing, ft
D = effective depth of footing, ft
d'' = Thickness of protective layer at bottom of footing
= min. 3 in. in reinf. concrete footings
= min. 2 in. in plain concrete footings

Other symbols (k, K, j, f_s, f_o, f_c', A_s, p, v, u, Σo, ϕ, etc.) are standard notations used in concrete design.

DERIVATION OF FORMULAS

Width of footing base for wall (Fig. 1)

$P + Q = P + q' (B - C) = qB$
$B(q - q') = P - q' c$

$$B = \frac{P - q' c}{q - q'} = c \times \left[\frac{\frac{P}{Q} - q'}{q - q'}\right] \quad \ldots\ldots (1)$$

if $q - q' = w_t$ net pressure (bending load)

$$B = C\left[\frac{f_o - q'}{w_t}\right] \text{ where } f_o = \frac{P}{C} \text{ tons per sq ft}$$

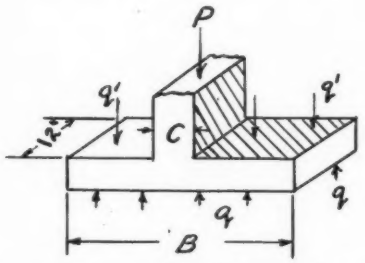


Fig. 1—Forces affecting width of wall footing

Width of base for column footing (Fig. 2)

$P + Q = P + q' (B_1 B_2 - c^2) = qB_1 B_2$
Let $B_2 = \alpha B_1$
$\alpha B_1^2 (q - q') = P - q' c^2$
$B_2 < B_1 \quad \alpha < 1$
For square footing $\alpha = 1$

$$B_1 = \sqrt{\frac{P - q' c^2}{\alpha (q - q')}} = c\sqrt{\frac{\frac{P}{c^2} - q'}{\alpha q - q'}}$$

$$= c\sqrt{\frac{f_o - q'}{\alpha w_t}} \quad \ldots\ldots (2)$$

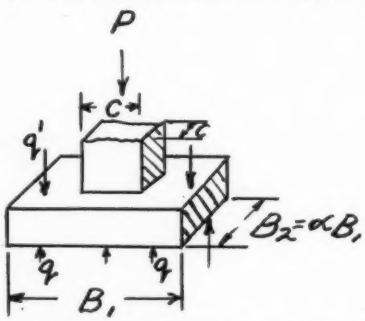


Fig. 2—Forces affecting width of column base

Thickness of footing required to resist bending moment

Wall footing—(All calculations refer to 1 ft length of footing.) Appropriate flexural coefficients are selected from Table 1. Table 2 lists unit stresses.

$$M = \frac{2000\, w_t L^2}{2} = \frac{(12D)^2}{6} f_c \text{ for plain concrete} \quad \ldots\ldots (3)$$

$$1000\, w_t L^2 = (12D)^2 K \text{ for reinforced concrete} \quad \ldots\ldots (4)$$

where $K = f_s pj$ $\ldots\ldots (5)$

$$k = \sqrt{2np + (np)^2} - np \quad \ldots\ldots (6)$$

TABLE 1—FLEXURAL COEFFICIENTS FOR WALL FOOTINGS*

P	$f_c' = 2000$ $n = 15$				$f_c' = 2500$ $n = 12$	$f_c' = 3000$ $n = 10$			
	k	K	Δ_1 †, $f_s = 18{,}000$	Δ'_1, $f_s = 20{,}000$	k	k	K	Δ_1, $f_s = 18{,}000$	Δ'_1, $f_s = 20{,}000$
0.001	0.159	17.0	0.640	0.608	0.143	0.132	17.2	0.635	0.603
0.002	0.217	33.4	0.456	0.433	0.196	0.181	33.8	0.453	0.430
0.003	0.259	49.3	0.376	0.357	0.235	0.217	50	0.372	0.353
0.004	0.292	65	0.327	0.311	0.266	0.246	66	0.324	0.308
0.0055	0.333	89	0.280	0.266	0.305	0.281	90	0.278	0.264
0.007	0.365	111	0.250	0.238	0.334	0.311	113	0.248	0.236
0.010	0.418	155	0.212	0.201	0.384	0.358	158	0.210	0.200
0.012	0.446	184	0.195	0.185	0.412	0.384	188	0.192	0.183

*Since values of Δ_1 for $f_c' = 3000$ differ from those for $f_c' = 2000$ by not more than 1 percent, the same set of D/L curves may be safely used for $f_c' = 2000$, 2500, 3000 and 3750.

Values of coefficient Δ_2 for column footings are $= \Delta \sqrt{0.85} = \frac{12}{13}\Delta$, (See ACI Building Code, par. 1204e.)

†For any other values of f_s coefficients Δ_1 and Δ_2 will vary in proportion to $\sqrt{\frac{18{,}000}{f_s}}$ (See Eq. 5 and 10).

For $f_s = 20{,}000$; $\sqrt{\frac{18}{20}} = 0.9487 = 0.95$

TABLE 2—COMPRESSIVE UNIT STRESS IN CONCRETE FOR VARYING STRENGTH OF CONCRETE AND STEEL*

P	$f_c' = 2000$		$f_c' = 2500$		$f_c' = 3000$	
	f_s, psi		f_s, psi		f_s, psi	
	18,000	20,000	18,000	20,000	18,000	20,000
0.001	226	252	252	280	272	302
0.002	332	368	368	409	398	442
0.003	418	465	460	511	500	555
0.004	493	548	540	600	585	650
0.0055	600	667	650	722	704	782
0.007	690	766	755	838	810	900
0.010	860	955	938	1040	1005	1116
0.012	970	1080	1050	1166	1125	1250
Max. f_c	900		1125		1350	

*$f_c = 2f_s \frac{p}{k} = 36{,}000 \frac{p}{k}$ (See Eq. 8) for $f_s = 18{,}000$ and 20,000 psi

$$j = 1 - \frac{k}{3} \quad \text{.....(7)}$$

$$f_c = f_s \frac{2p}{k} \quad \text{.....(8)}$$

Hence:

$$\frac{D}{L} = \frac{1}{12}\sqrt{\frac{6000 w_t}{f_c}} \text{ for plain concrete} \quad \text{.....(9)}$$

$$\frac{D}{L} = \frac{1}{12}\sqrt{\frac{1000}{K}\, w_t} = \Delta_1 \sqrt{w_t} \text{ for reinforced concrete} \quad \text{.....(10)}$$

Column footing—In 2-way reinforced footings 0.85 M is substituted for M in Eq. 3 and 4. Since $\sqrt{0.85} = \frac{12}{13}$ denominator 12 in Eq. 9 and 10 is changed to 13.

$$\frac{D}{L} = \frac{1}{13}\sqrt{\frac{6000}{f_c}\, w_t} \text{ for plain concrete} \quad \text{.....(11)}$$

$$\frac{D}{L} = \frac{1}{13}\sqrt{\frac{1000}{K}\, w_t} = \Delta_2 \sqrt{w_t} \text{ for reinforced concrete} \quad \text{.....(12)}$$

Thickness of footing required to resist shear

Wall footing—Unit shear stress (Fig. 3)

$$v = \frac{2000\, w_t\, (L - D)}{144 \times 7/8\, D}$$

$$\text{If } \delta = \left(\frac{D}{L}\right);\left(\frac{v}{w_t}\right) = 15.9\,\frac{1-\delta}{\delta} \quad \ldots (13)$$

$$\delta\,(v + 15.9\, w_t) = 15.9\, w_t$$

$$\delta = \frac{1}{1 + \dfrac{v}{15.9\, w_t}} \quad \ldots (14)$$

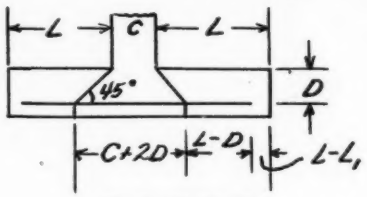


Fig. 3—Wall footing

Column footing (Fig. 4)

$$v = \frac{2000\, w_t}{144} \times \frac{0.5\,[(C + 2D) + (C + 2L)] \times (L - D) - (L - L_1)^2}{7/8D\,(C + 2D)}$$

$$v = 15.9\,\frac{(C + D + L)\,(L - D) - (L - L_1)^2\, w_t}{(C + 2D)\, D} \quad \ldots (15)$$

$$\frac{C}{L} = \gamma;\frac{D}{L} = \delta;\frac{L_1}{L} = \lambda \qquad \delta < \lambda < 1$$

$$\frac{v}{w_t} = 15.9\,\frac{(\gamma + \delta + 1)\,(1 - \delta) - (1 - \lambda)^2}{(\gamma + 2\delta)\,\delta} \quad \ldots (16)$$

For square footing

$$L_1 = L \qquad \lambda = 1 \qquad \left(\frac{v}{w_t}\right) = 15.9\left[\frac{(1 + \gamma + \delta)\,(1 - \delta)}{(\gamma + 2\delta)\,\delta}\right] \quad \ldots (17)$$

When $L_1 < D$ load area becomes rectangular and Eq. (13) is used (Fig. 5).

Bond stress and its relation to diameter of bars

$\Sigma\, 0$ = Sum of bar perimeters per 1 ft width, in.
ϕ = Bar diam., in. A_s = Area of steel, sq in per ft
b = 12 in. m = number of bars in 1 ft width
u = Bond unit stress (Table 3)
v_o = Shear unit stress used as a measure of bond (ACI Building Code, par. 1205d)

$$u\,\Sigma\, 0 = v_o b \quad \ldots (18)$$

$$\Sigma\, 0 = \frac{v_o b}{u} = 12\,\frac{v_o}{u} \quad \ldots (19)$$

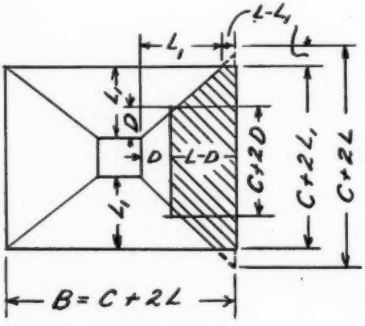


Fig. 4 (Left)—Column footing

Fig. 5 (Below)—Rectangular footing

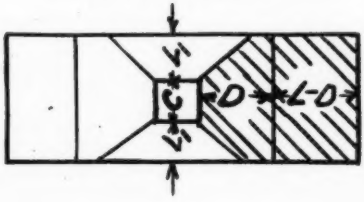


TABLE 3—ALLOWABLE BOND UNIT STRESS, u

	$f_c' = 2000$	$f_c' = 2500$	$f_c' = 3000$
Boston Code	200	200	200
ACI Code			
1-way reinforcement	150	188	225
2-way reinforcement	112	140	168

$$v_o = \frac{w_t \times 2000\ \mathrm{L}}{7/8\ D \times 144} \qquad \left(\frac{D}{L}\right) = \delta$$

$$\Sigma\, 0 = \frac{2000 \times 12\ w_t}{7/8 \times 144\ u\ \delta} = \frac{190}{u} \times \left(\frac{w_t}{\delta}\right) = C \times \left(\frac{w_t}{\delta}\right) \qquad (20)$$

In a 2-way column footing $v_o' = 0.85\ v_o$. Since v_o differs considerably from v and requires separate computation, it is often more convenient to use the following Eq. which give directly maximum bar diameter as function of $\left(\frac{\delta}{W_t}\right)$.

$$\Sigma\, o = m\ \pi\ \phi \text{ for round bars} = m\ 4\ d \text{ for square bars} \qquad (21)$$

$$A_s = m\ \pi\ \frac{\phi^2}{4} \text{ for round bars} = m\ d^2 \text{ for square bars}$$

$$A_s = \Sigma\, o \times \frac{\phi}{4} \qquad (22)$$

$$\left(\frac{\phi}{A_s}\right) = \frac{4}{\Sigma\, 0}. \text{ Substituting } \Sigma 0 \text{ from Eq. 19; } \left(\frac{\phi}{A_s}\right) = \frac{u}{3v_o} \qquad (23)$$

Substituting $\Sigma\, 0$ from Eq. 20 we get a simple formula easy to tabulate:

$$\left(\frac{\phi}{A_s}\right) = \frac{4u}{190} \times \left(\frac{\delta}{w_t}\right) = \frac{4}{c} \times \left(\frac{\delta}{w_t}\right) \qquad (24)$$

Theoretical diameter of bars, ϕ, (we might call it "selective," "preferable," "proper" or "ideal") satisfies two requirements: Ideal steel area and bar perimeters correspond exactly to assumed *tension and bond* unit stresses (Table 4). If larger diameter is chosen and given bond stress is not to be exceeded total area, A_s, and percentage, p, of steel must be increased in the same proportion to obtain required perimeter $\Sigma 0$ (See Eq. 23); and, if footing depth remains unchanged this means lower tension stress in steel.

In the opposite case, if because of code limitations of minimum depth (AASHO—2 or 15 in. above piles; ACI—6 or 12 in. above piles for reinforced

TABLE 4—IDEAL PERIMETER AND DIAMETER OF ROUND STEEL BARS

	$\Sigma\, O$		ϕ/A_s	
u	For walls	For columns	For walls	For columns
112		$1.44\left(\frac{w_t}{\delta}\right)$		$2.78\left(\frac{\delta}{w_t}\right)$
140		$1.15\left(\frac{w_t}{\delta}\right)$		$3.48\left(\frac{\delta}{w_t}\right)$
150	$1.27\left(\frac{w_t}{\delta}\right)$		$3.15\left(\frac{\delta}{w_t}\right)$	
188	$1.01\left(\frac{w_t}{\delta}\right)$		$3.95\left(\frac{\delta}{w_t}\right)$	
200	$0.95\left(\frac{w_t}{\delta}\right)$	$0.81\left(\frac{w_t}{\delta}\right)$	$4.20\left(\frac{\delta}{w_t}\right)$	$4.94\left(\frac{\delta}{w_t}\right)$

concrete or 8 or 14 in. on piles for plain concrete, minimum $p = 0.25$ percent) the steel area used is greater than required and shown on the chart, then the diameter of bars may be increased in the same proportion.

DESCRIPTION OF CHARTS

Three kinds of curves are shown in Fig. 6-10. These are D/L curves for design based on bending, for design based on shear, and v/w curves for checking shear.

Fig. 6-9 show depth-to-span ratio, D/L, as a function of (a) net pressure, w_t, (b) percentage of steel reinforcement, p,* and (c) compressive unit stresses in concrete and tension in steel for $f_s = 18{,}000$ psi and $f_s = 20{,}000$ psi.† (See Eq. 9-12.) In both cases, whether 1-way or 2-way reinforcement, p refers to the 1-way steel quantity.

These curves serve three purposes: (1) designing depth, D, for given load, w_t, (2) checking actual stresses under given load, w_t, and (3) figuring maximum soil pressure that may be allowed for given type of footing and given unit stresses. The tables at the bottom of Fig. 6 and 7 give diameter of bars for various values of allowable bond stress. Fig. 10 gives the depth-to-span ratio as a function of (a) net pressure, w_t, (b) allowable shear unit stress, v, and (c) ratio of span to column width, L/C (See Eq. 14-17). There are three curves for wall footings for limiting values of $v = 40$, 60 and 70 psi and two sets of curves (three to each set) for square column footings with $L = C$ and $L = 3C$. These curves with Table 5 cover almost all cases of column footings that are likely to be met in present practice.

The extreme case $L = 4C$ can happen only when average compression unit stress in a high grade concrete column has maximum allowable value and net pressure is at its lowest limit.

Example 1

According to the ACI Code average axial compression unit stress in column

*Most economical value of p depends on relative prices of steel and concrete. For formula see p. 346.
†See third footnote, Table 1.

TABLE 5—CORRECTION COEFFICIENTS FOR SHEAR STRESS, v, IN RECTANGULAR COLUMN FOOTINGS

$\gamma = \frac{C}{L}$	$\delta = \frac{D}{L}$	$\lambda = \frac{L_1}{L}$ 0.75	0.66	0.50
0.3	0.3	0.95	0.90	0.78
	0.5	0.93	0.88	0.72
0.5	0.3	0.95	0.91	0.80
	0.5	0.94	0.89	0.75
	0.7	0.91	*	*
0.7	0.5	0.95	0.90	0.77
	0.7	0.92	*	*

*Fig. 6 and 7 with rectangular load area should be used in these cases because $L_1 \leqq D$; $\lambda \leqq \delta$.

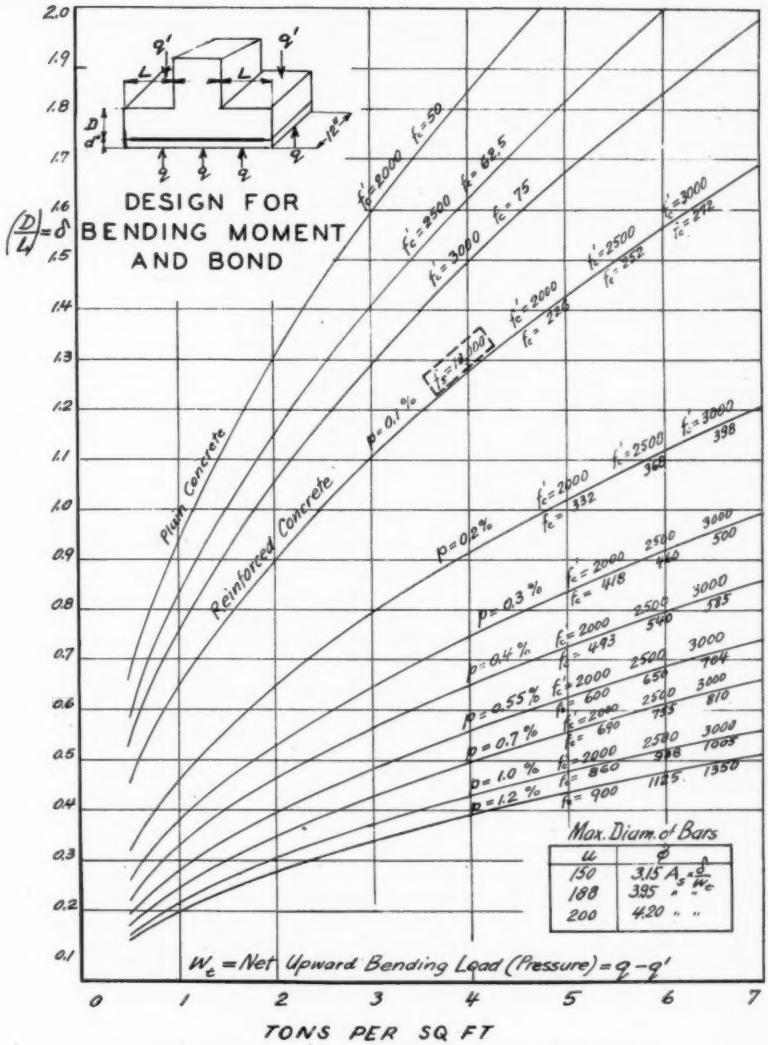


Fig. 6—Concrete wall footings under axial load, f_c = 18,000 psi

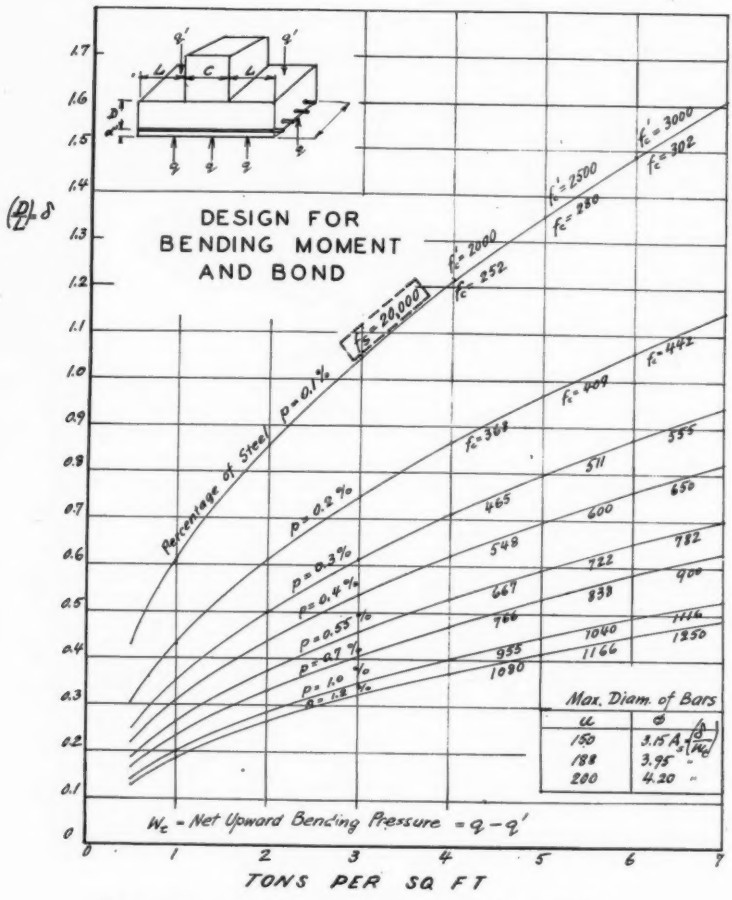


Fig. 7—Concrete wall footings under axial load, f_s = 20,000 psi

is computed as follows:

Max. $f_a = 0.25 \times 3000\ (1 + (n - 1)p)$ $\quad n = 10$
Assume $p = 1.25$ percent, then max. $f_a = 830 = 60$ tons per sq ft
Assume soil pressure q as low as 1 ton per sq ft and an extra heavy surcharge on footing:

Warehouse live load	200
Floor and footing (2 ft)	300
Fill (1 ft)	100
Total	$q' = 600$ $\quad = 0.3$ tons per sq ft

Min. net pressure $w_t = 1 - 0.3 = 0.7$ tons per sq ft

$$B = C\sqrt{\frac{60}{0.7}} = 9C$$

$$L = 0.5\ (B - C) = 0.5\ (9C - C) = 4C$$

When $w_t = 1.23$ tons per sq ft

$$B = C\sqrt{\frac{60}{1.23}} = 7C,\ L = 0.5\ (7 - 1)\ C = 3C$$

Thus, the case $L = 4$ can emerge only between values of $w_t = 0.7$ and 1.23 tons per sq ft. Within these limits the D/L curve for $L = 4$ nearly coincides with that for $L = 3$ (Fig. 10). Hence, the latter may be safely used when $L = 4C$.

Curves for checking shear unit stress

The ratio v/w_t (on upper horizontal scale, Fig. 10) is given as function of D/L (on vertical scale) for wall footings (Eq. 13) and 3 curves for ratio v/w_t (on upper horizontal scale) as function of D/L for values of $L = C$; $L = 3C$ and $L = 4C$ for column footings (Eq. 16).

Value of v/w_t multiplied by given w_t in tons per sq ft gives value of v in psf.

Rectangular column footings

Figs. 8 and 9 may also be used for rectangular column footings. Greatest actual shear in direction of length of footing is a little less than that in square footing. Reduction coefficients for v/w_t for rectangular footings calculated as ratios of the figures obtained from Eq. 16 and 17, respectively, are given in Table 5.

Since required D/L value for rectangular footings cannot be determined from D/L curves prepared for square footings, cut-and-try methods must be used. After preliminary value of D/L ratio has been found from the diagram, shear unit stress, v, given by v/w_t curve should be multiplied by correction coefficient taken from Table 5. If this is not satisfactory, next approximation will give the required solution.

TYPICAL EXAMPLES

Example 2—Plain concrete wall footing*

$f_c' = 2000$, $L = 3$, $C = 1.5$, $w_t = 4$ tons per sq ft
From Fig. 6 for $f_c' = 2000$, $D/L = 1.83$

*See Example 1 for computation of width of base.

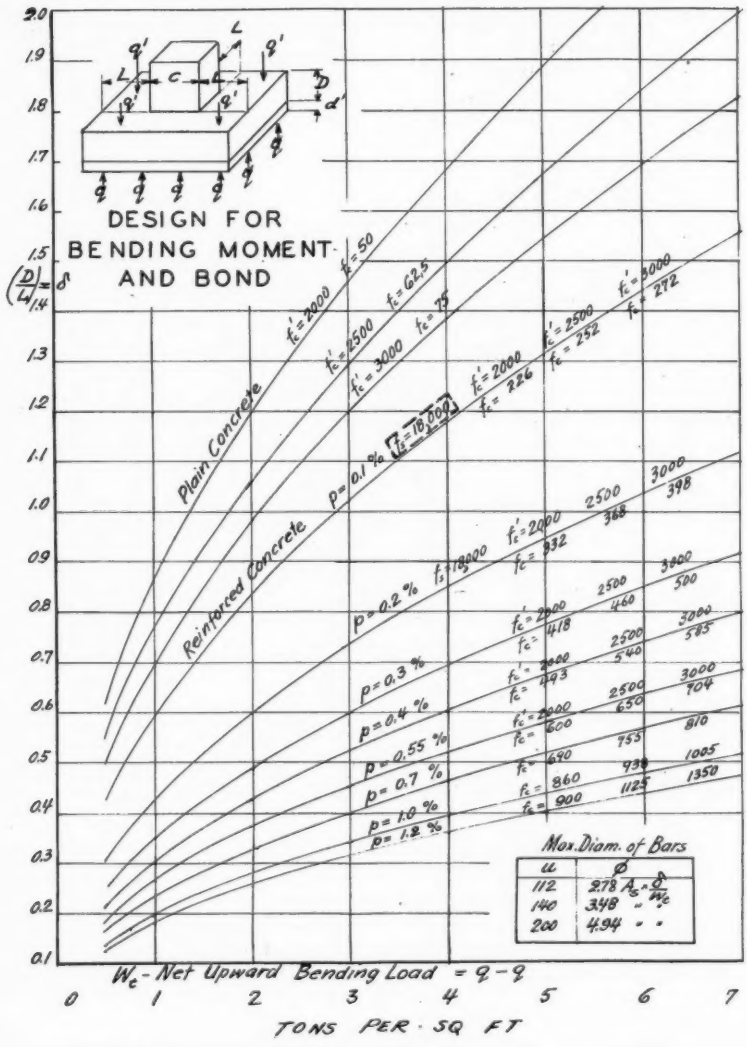


Fig. 8—Square concrete column footings under axial loads, f_s = 18,000 psi

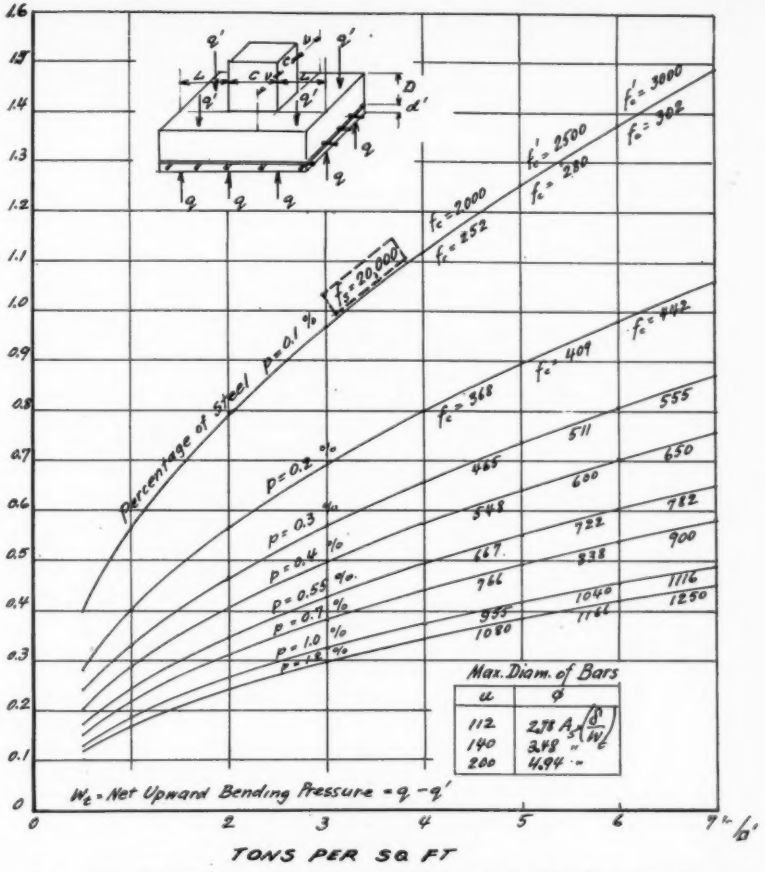


Fig. 9—Square reinforced concrete column footings under axial load, f_s = 20,000 psi

Hence $D = 1.83 \times 3 = 5.5$ ft = 5 ft 6 in.

d'', min.	2 in.
Total depth	5 ft 8 in.

Since $D > L$ no check for shear stress is required.

Example 3—Reinforced concrete square column footing

$f_c' = 2500$, $v = 75$. w_t, L and C as before. $L/C = 3/1.5 = 2$, $L = 2C$, $w_t = 4$

Depth of Concrete—D/L curves in Fig. 10 based on shear, $v = 75$, for $L = C$ and $L = 3C$ give (by interpolation for $L = 2C$) for $w_t = 4$

Min. $D/L = 0.524$ $D = 0.524 \times 3 = 1.572$ ft = 1 ft 7 in.

d," min.	3 in.
Total depth =	1 ft 10 in.

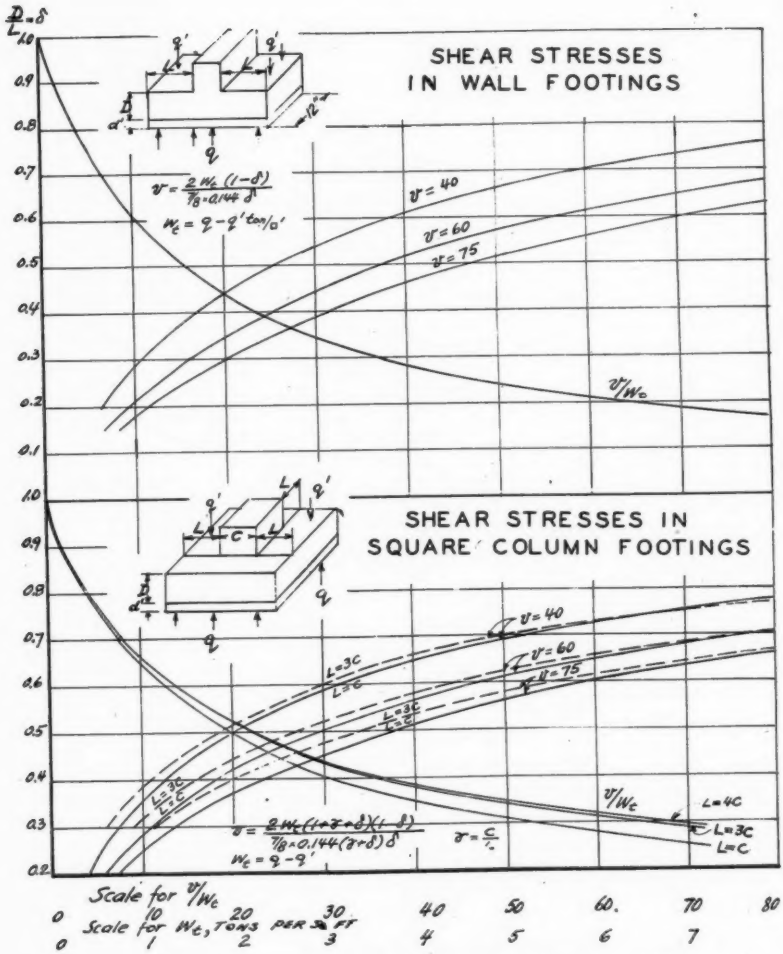


Fig. 10—Shear stresses in concrete wall and column footings under axial loads

Steel—Fig. 6 shows that point for $w_t = 4$ and $D/L = 0.524$ falls very close to the curve of required percentage of steel, $p = 0.0055$.

Steel required, $A_s = pD = 0.0055 \times 1.572 \times 144 = 1.245$ sq in. per ft.

Compression unit stresses in concrete—For $f_c' = 2000$, $f_c = 600$. For $f_c' = 2500$, $f_c = 650$.

In any case, with the low percentage of steel used in concrete footings it is not compression but shear in concrete as well as tension in steel and bond which govern design.

For $f_s = 20{,}000$; $D/L = 0.524\sqrt{\frac{18}{20}}$ (See Note 3, Table 1)

If larger thickness, D, has been selected, say $D/L = 0.625$, then point of intersection of this horizontal with vertical cutting lower scale at $w_t = 4$ falls between curves $p = 0.003$ and 0.004 (Fig. 8). By interpolation $p = 0.00375$. $A_s = 0.00375 \times 0.625 \times 144 = 1.0$ sq in.

Now to find actual value of shear unit stress, v, a horizontal line is drawn through point on vertical scale $D/L = 0.625$ to intersection with v/w_t curves for $L = C$ and $L = 3C$ (Fig. 10). Vertical dropped from this interpolated point cuts upper horizontal scale at point $v/w_t = 1.16$. Hence, if $w_t = 4$ tons per sq ft, $v = 11.6 \times 4 = 46.4$ psi.

Bond stress—According to the ACI Code for 2-way reinforced concrete footings with $f_c' = 2500$, maximum $u = 140$. Theoretically proper diameter of bars, ϕ, is given in the table at bottom of Fig. 8.

$$\phi = 3.95\, A_s \times \frac{\delta}{w_t} \qquad A_s = 1.01 \qquad \delta = 0.625 \qquad w_t = 4$$

Hence $\phi = 3.95 \times 1.01 \times 0.625/4 = 0.624 = \frac{5}{8}$ in.

Example 4—Rectangular footing

Design for bending and bond is the same as that described for a square footing. Only design for shear requires different method.

Let $L_1 = \frac{2}{3}L$, $L = 3C$, $\gamma = \frac{C}{L} = 0.33$, $w_t = 4$, $v = 75$

D/L was found for square footing to be $\delta = 0.524$

From Table 5 reduction coefficient = 0.88 ±. Find on D/L curve, $L = 3C$ and $v = 75$ (Fig. 10) value of D/L corresponding to $w_t = 0.88$. $W = 0.88 \times 4 = 3.52$. This value is $\delta = 0.508$. Now, v/w_t curve ($L = 3C$) gives $v/w_t = 21$ for $D/L = 0.508$. Hence, $v = 21 \times 4 \times 0.88 = 74$, which is just satisfactory.

ECONOMICAL PERCENTAGE OF STEEL IN CONCRETE

As long as steel unit stress is kept constant, quantities of concrete and reinforcing steel are mutually related by a law represented by Eq. 4 and 5: $1000\, wL^2 = 144\, D^2 f_s pj$;

$$pD \times D = \frac{1000\, wL^2}{144\, f_s j} \qquad (29)$$

When quantity of steel, pD, increases, quantity of concrete, D, decreases. In a similar way the cost of two materials varies. (See example p. 347)

j varies very little and for practical purposes is usually considered constant. Indeed, within the range of p used in footings 100 percent increase of p produces only 2 to 4 percent variation of j. Hence, with sufficient approximation Eq. 29 may be represented as follows:

$$D^2 p = C \text{ (constant)} \quad (30)$$

$$D = \sqrt{\frac{C}{p}} \quad (31)$$

If a = price of 1 cu ft of steel
b = price of 1 cu ft of concrete (including forms)

Then *total cost* of 1 sq ft of reinforced concrete footing will be

$$pDa + Db = (ap + b)D \quad (32)$$

Substitute D in terms of p from Eq. 31

$$(ap + b)\sqrt{\frac{c}{p}} = \text{f } p \quad (33)$$

The total cost becomes minimum when $f' p = 0$

$$a\frac{\sqrt{C}}{\sqrt{p}} - (ap + b)\frac{\sqrt{C}}{2\sqrt{p^3}} = 0 \quad (34)$$

$$2ap - ap = b \qquad ap = b \qquad p = \frac{b}{a} \quad (35)$$

If cost of 1 lb of steel = a_1 dollars per lb and cost of 1 cu yd of concrete = b_1 dollars per cu yd

$$a = 490\, a_1$$
$$b = \frac{b_1}{27}$$

$$\text{Then } p = \frac{1}{490 \times 27} \times \frac{(b_1)}{(a_1)} = \frac{1}{132} \times \frac{(b_1)}{(a_1)} \text{ percent} \quad (36)$$

When both materials are to be used efficiently for the given loads and stresses, the following conclusions may be drawn:

1. p corresponding to minimum total cost is *independent of loading and stresses*, in concrete as well as in steel.

2. p depends only on price ratio. b/a = unit cost of concrete (including forming and placing)/unit price of steel (including bending and placing).

3. p varies in direct proportion to unit price of concrete and inversely proportional to the unit price of steel. Ten percent higher price of steel would lower p to 0.9 of its current value (1.2 percent in May 1950, in Boston). Five percent lower price of concrete will raise p to 1.05 of its present value.

4. p in all cases denotes total percentage of steel. In 1-way reinforced footings p includes spacing bars also. If in 2-way reinforced footing we assume primarily p_1 = percentage of 1-way steel (which is consistent with basic Eq. 30), then Total cost (Eq. 32) = $2p_1 Da + Db$. But $2p_1 D \times a = p_1 D$

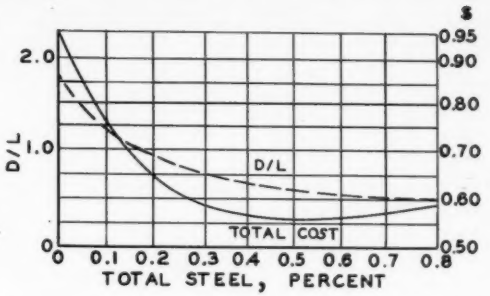


Fig. 11—Variation of cost with percentage of steel in concrete

$\times 2a$ which means that final Eq. 35 will give for 1-way steel or for ½ of the total steel

$$p_1 = \frac{b}{2a} = \frac{1}{2} \times \frac{b}{a} = \frac{p}{2}.$$

Fig. 11 shows how cost varies with changes in percentage of steel and D/L.

Example 5—2-way reinforced column footing (Table 6)

Where $f_o = \dfrac{P}{c^2}$ tons per sq ft

f_s = 18,000
f_c' = 2000
w_1 = 4 tons per sq ft
L = 1 ft

Price of steel
a_1 = 0.09 per lb
$2a = 490 \times 0.09 \times 2$
= 88

Price of concrete
C_1 = 14 per cu yd

$C = \dfrac{14}{27} = 0.52$

TABLE 6—COST DATA FOR REINFORCED CONCRETE

p per cent	D/L	Cost of 2-way Steel $2ap\frac{D}{L} = 0.88p\frac{D}{L}$	Cost of concrete $0.52\frac{D}{L}$	Total cost per sq ft
0	1.83	0	0.95	0.95
0.1	1.28	0.113	0.66	0.77
0.2	0.92	0.162	0.48	0.64
0.3	0.75	0.198	0.39	0.58
0.4	0.65	0.230	0.34	0.57
0.55	0.56	0.272	0.29	0.56
0.7	0.50	0.308	0.26	0.57
0.8	0.472	0.333	0.245	0.58

Total cost is minimum when $p = \dfrac{b}{2a} = \dfrac{0.52}{0.88} = 0.59$ percent or $p = \dfrac{14}{132 \times 0.09} = 0.59$ percent

ECCENTRICALLY LOADED FOOTINGS

Width of footing

Wall footings—(Fig. 12)

$$\text{Max. } q = \frac{P}{B} + \frac{6eP}{B^2} + q' \frac{(B - C)}{B} \quad \ldots\ldots (37)$$

Multiply by B^2

Max. $qB^2 - PB - 6eP - q'B^2 + q'CB = 0$

B^2 (max. $q - q'$) $- B\,(P - q'c) - 6eP = 0$

$$B^2 - B\left[\frac{P/c - q'}{\text{max. } q - q'}\right] c - \frac{6\,e/c \times P/c}{\text{max. } q - q'} c^2 = 0 \quad \ldots\ldots (38)$$

$\dfrac{P}{C} = f_o$, average compression unit stress in column

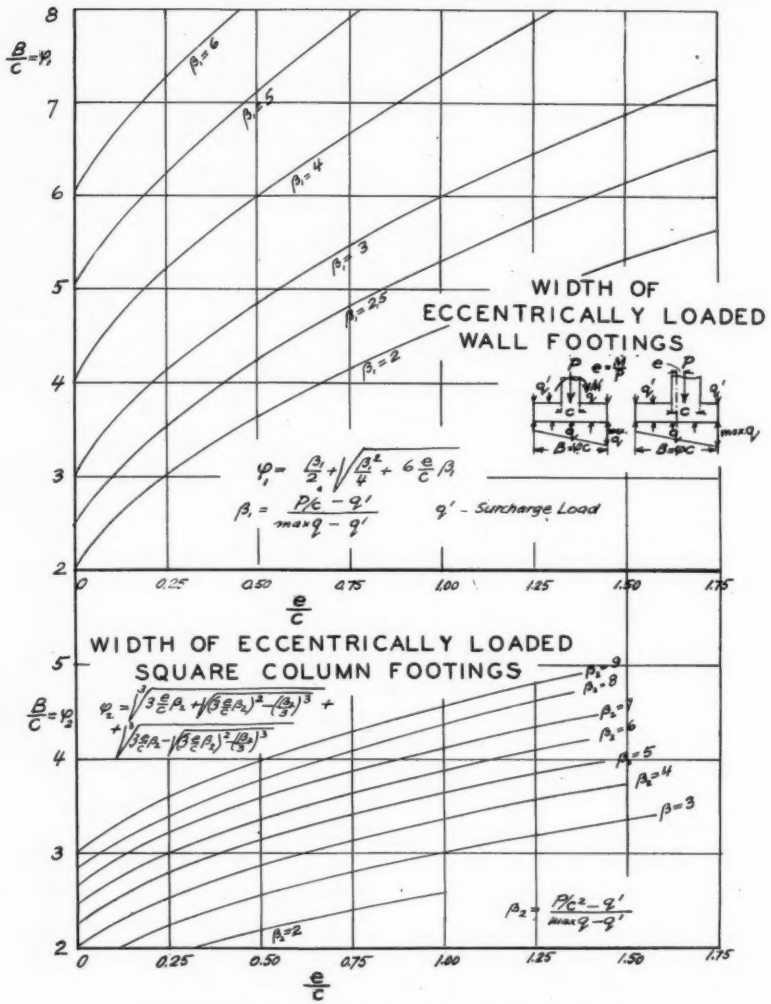

WIDTH OF ECCENTRICALLY LOADED WALL FOOTINGS
WIDTH OF ECCENTRICALLY LOADED SQUARE COLUMN FOOTINGS
q' - Surcharge Load
e/c

Fig. 12—Width of eccentrically loaded wall and column footings

q' being very small (250-700 tons per sq ft) in comparison with P/c (250-1500 psi) may be neglected.

$$B^2 - \beta_1 cB - 6\, e/c\, \beta_1 c^2 = 0 \quad \ldots\ldots (39)$$

$$\beta_1 = \left[\frac{P/c}{\text{max. } q - q'}\right] \quad \ldots\ldots (40)$$

$$B = C\left[\frac{\beta_1}{2} + \sqrt{\frac{\beta_2}{4} + 6\, e/c\, \beta_1}\right] = \varphi\, c \quad \ldots\ldots (41)$$

Example 6

Wall width, $c = 2$ ft, $P = 30$ tons, $e = 1$ ft, $e/c = 0.5$ max. $q = 4$ tons per sq ft and $q' = 0.6$ tons per sq ft

$$\beta_1 = \frac{30/2 - 0.6}{4.0 - 0.6} = 4.25$$

Interpolation between curves $\beta_1 = 4$ and 5 (Top of Fig. 12) gives $\varphi = 6.27$ for $e/c = 0.5$. Hence, $B = 6.27 \times 2 = 12.54$ ft

Square column footings

$$\text{Max. } q = \frac{P}{B^2} + \frac{6eP}{B^3} + q\frac{(B^2 - c^2)}{B^2} \quad \ldots\ldots (42)$$

$$B^3 - \left[\frac{P - q'\, c^2}{\text{max. } q - q'}\right] B - \frac{6eP}{\text{max. } q - q'} = 0 \quad \ldots\ldots (43)$$

$$\left[\frac{\frac{P}{c^2}}{\text{max. } q - q'}\right] = \beta_2$$

$$B^3 - \beta_2 c^2 B - 6\, e/c\, \beta_2 c^2 = 0 \quad \ldots\ldots (44)$$

q' is negligible in comparison with P/c^2

$$B = c\left[\sqrt[3]{3e/c\beta_2 + \sqrt{(3e/c\beta_2)^2 - \left(\frac{\beta_2}{3}\right)^3}} + \sqrt[3]{3e/c\beta_2 - \sqrt{(3e/c\beta_2)^2 - \left(\frac{\beta_2}{3}\right)^3}}\right] = \varphi_2\, c \quad \ldots\ldots (45)$$

Example 7

Data as for previous example. Column dimensions 2 x 2 ft

$\beta_2 = \dfrac{30/4 - 0.6}{4.0 - 0.6} = 2.025$ (See chart for φ_2, bottom of Fig. 12)

φ — curve for $\beta_2 = 2$ gives $\varphi_2 = 2.2$ for $\dfrac{e}{c} = 0.5$

Hence $B = 2.2 \times 2.025 = 4.45$ ft

Rectangular footing under single column

If lateral dimension of footing is assumed as αB where B is width of base in main direction, *viz.* direction of eccentricity, then the area of the base is αB^2. If we substitute this expression instead of B^2 in basic Eq. 42 for maximum q under square footing eccentrically loaded we get

$$\text{Max } q = \frac{P}{\alpha B^2} + \frac{6eP}{\alpha B^3} + q'\frac{(B - c^2)}{\alpha B^2} \quad \ldots\ldots (46)$$

multiplying by αB^3

$$\text{Max } q\, \alpha B^3 = PB + 6eP + q' B^3 - q' c^2 B$$

$$B^3 (\alpha q_m - q') - (P - q' c^2) B - 6eP = 0 \quad \ldots\ldots (47)$$

$$B^3 - \left[\frac{P - q' c^2}{\alpha q_m - q'}\right] B - \frac{6eP}{\alpha q_m - q'} = 0 \quad \text{.....(48)}$$

It is the same equation as in case of square footing except for the modified value of the constant.

$$\beta_2 = \left[\frac{P/c^2}{\alpha q_m - q'}\right] \quad \text{.....(49)}$$

Eccentric location of wall or column

As may be seen from the basic formulas for maximum soil pressure (max. q) in all cases (wall, square and rectangular footings) eccentricity is measured from center of footing base and not from that of wall or column. Hence, all the above formulas are equally applicable to cases where wall or column is located eccentrically in reference to footing center.

If a wall or column beside being eccentrically located is also eccentrically loaded, then e in the formulas means total eccentricity of load P referred to footing center.

Bending moment in eccentrically loaded footing

Bending load, or net pressure, $w = q_x - q'$ (where q' is surcharge load) is not distributed uniformly as in axially loaded footing but is uniformly decreasing from maximum under one edge to minimum at opposite side (Fig. 13).

Although very often, for sake of simplicity, such footings are designed for maximum w, pressure assumed uniformly distributed, in some cases economy may require more exact calculation of actual, reduced bending moment. The formula for the latter case is:

Call: max. $q = q_m$, q = average pressure, max. $w = w_m$
$q_m = q + \Delta q = (1 + \Delta)\, q$
$w_m = w_m - q'$

At face of wall or column.

$$w = \left(q_m - \Delta q \frac{L}{0.5}\right) - q' = (q_m - q') - \Delta q \frac{L}{0.5} = w_m - \Delta q \frac{L}{0.5} \quad \text{.....(50)}$$

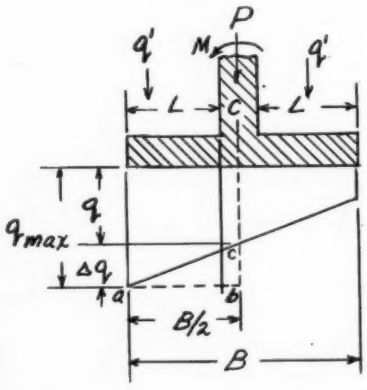


Fig. 13—Distribution of bending load in eccentrically loaded footing

$$BM = \frac{w_m L^2}{2} - \frac{\Delta q L}{0.5} \times \frac{L}{2} \times \frac{L}{3} = \frac{w_m L^2}{2}\left(1 - \frac{2}{3}\frac{\Delta q}{w_m} \times \frac{L}{B}\right) = \frac{\omega w_m L^2}{2} \quad \ldots\ldots (51)$$

$$\omega = \left(1 - \frac{2}{3}\frac{\Delta q}{w_m} \times \frac{L}{B}\right) \quad \ldots\ldots (52)$$

$$\Delta = \frac{6e}{B} \quad \ldots\ldots (53)$$

Thus, if instead of w_m expressed in tons = w_t we substitute w_m multiplied by reduction coefficient ω the same charts and the same procedure for dimensioning and reinforcing may be used as for axially loaded footings.

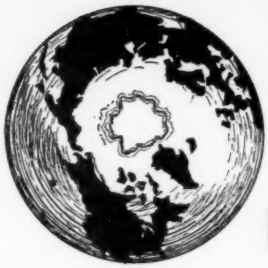
ACKNOWLEDGMENTS

The author is indebted and grateful to A. Russell Barnes, project manager and Jack Jarnis and Melvin Ja, assistant project managers, Thomas Worcester, Inc., for valuable suggestions and corrections to the paper.

REFERENCES

1. Taylor, Frederick W., Thompson, Sanford E. and Smulski, Edward, *Concrete Plain and Reinforced*, John Wiley and Sons, Inc., New York.
2. "Building Code Requirements for Reinforced Concrete (ACI 318-51)," American Concrete Institute, Detroit.
3. *Reinforced Concrete Design Handbook*, ACI Committee 317.

EDITOR'S NOTE: Readers will note that bond values used in this paper are those specified by Building Code Requirements for Reinforced Concrete (ACI 318-47) which was in effect when the paper was submitted. For use with the 1951 Code appropriate values must be substituted.



Letters from Readers*

BY WAY OF SYNOPSIS

E. L. Howard and George Leavitt discuss the comparative accuracy of the Kelly penetrator ball and the slump cone in measuring consistency of concrete.

Charles Macklin adds further information on anchor bolts in concrete footings.

Kelly Ball *vs.* Slump Cone—Comparative Tests of Samples Taken at Travis Air Base (LR 48-16)

Some time ago Prof. J. W. Kelly of the University of California introduced the ball method of measuring consistency of concrete. Since then we have taken every opportunity to test its efficiency under job conditions.

During the placing of concrete parking facilities at Travis Air Base the Corps of Engineers gave permission to conduct comparative consistency tests of concrete placed in slabs with the Kelly Ball and slump cone. We made no tests of the concrete other than these Kelly Ball readings and slumps, nor did we test materials beyond the car unloading point. For much of the data shown here we are indebted to the project engineer and his staff.

The concrete mix used at the time these tests were taken is shown in Table 1. Designed by the Corps of Engineers, it had a cement factor of six sacks and was to be placed at a 3-in. maximum slump. The aggregates were produced from two sources; all gravel came from an operation on the American River, near Fair Oaks, Calif., and the sand was produced at a deposit on Cache Creek not far from Yolo, Calif. The gravel was a hard, dense material with specific gravity of 2.82. The gradings of the individual sized materials used are shown in Fig. 1. These data are taken from tests run by Pacific Coast Aggregates, Inc., as the shipments were made. Each curve is the average of more than fifty samples.

The aggregates were shipped by rail, transferred to trucks by a clamshell bucket crane, then dumped into stockpile to be elevated as needed to the batching plant by a conveyor belt system. Normally all this handling would cause substantial changes in the grading analysis of the aggregates, particularly the coarser sizes. On this job,

TABLE 1—MATERIALS TO MAKE ONE CU YD OF CONCRETE

Material	Weight, lb	Remarks
Cement, Type I	564	
Sand	1040	Incl. 5 percent moisture
Gravel		
¾ to ¼ in.	660	No moisture
1½ to ¾ in.	817	No moisture
2½ to 1½ in.	980	No moisture
Water	190	

*A part of copyrighted Journal of the American Concrete Institute, V. 23, No. 4, Dec. 1951, *Proceedings* V. 48. Separate prints of the entire Letters from Readers Section are available at 35 cents each. Address 18263 W. McNichols Rd., Detroit 19, Mich.

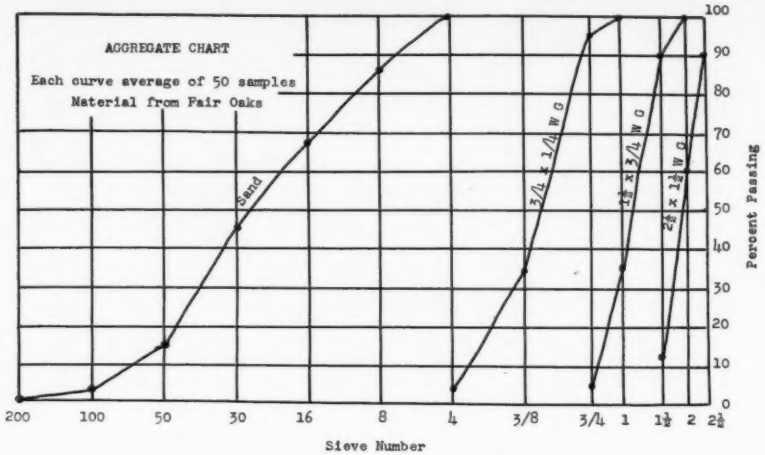


Fig. 1—Four grading curves show averages for material used for concrete on Travis Air Base

however, aggregates were re-screened by a vibrating screen atop the batching plant to remove the breakage. Thus the aggregates batched into concrete were graded substantially the same as when shipped.

The specifications for flexural strength, 3-point loading, at 14 days was 550 psi. Table 2 shows data on the strength of concrete specimens made when these consistency tests were run.

These data were furnished by the project engineer, Ellsworth M. Kanzler whose staff made all such strength tests mentioned here. Donald D. Fenton, construction engineer, chose location and time of the tests. With his fine cooperation it was possible to get samples that were fair and advantageous to the slump cone and the Kelly Ball. Lewis F. Pease, materials engineer, and George Leavitt, Pacific Coast Aggregates, Inc., made the tests. To insure the best and fairest results, these men alternated taking slumps and ball readings.

The Kelly Ball (described in detail in ASTM *Bulletin* No. 163, Jan. 1950) weighs 30 lb and is 6 in. in diameter (Fig. 2). Consistency of the concrete is measured in terms of the penetration of the ball in the fresh concrete. This penetration is read from the scale on the handle of the ball; the recorded penetration usually is the average of two or three readings. The ball may be used in forms, buggies, on the slab, or any other convenient place.

On this job the Kelly Ball readings were made on concrete on the grade just after

TABLE 2—FLEXURAL STRENGTH OF CONCRETE

Age in days	3	14
Number of samples	6	12
Average psi	436	656
Standard deviation	30	161

Fig. 2—Measuring consistency with a Kelly Ball

Fig. 3—Measuring slump of sample taken from batch of paving concrete

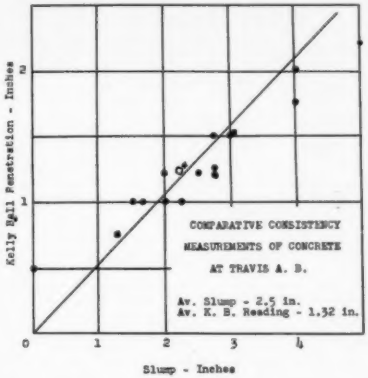


Fig. 4—Comparative consistency measurements made with slump cone and Kelly Ball of paving concrete at Travis Air Base

being mixed and dumped by the paver and before being worked by the spreader and screed (Fig. 2). The slumps were made using material sampled at the same time and from the same location as the concrete tested with the ball (Fig. 3). All slumps were taken in a manner conforming to ASTM C 143-39.

To take a slump required ten minutes, about the time necessary for the screed to reach the spot where the sampled concrete had been dumped by the paver. The taking of a slump involved securing a representative sample from the batch discharged from the mixer, wetting down the slump cone and base board, making the test in the standard method, and washing all equipment. In comparison, only ten seconds were required to make a Kelly Ball test. No delay whatever could be caused operations while this test was made. A flat spot on the freshly piled concrete was made with a shovel and the ball set on two places. The average of these two readings was recorded as the penetration. Clean up of equipment was done simply by wiping the dampness from the ball with a cloth. So quick and simple was the test that the mixerman could be advised of any desired change in water before a second batch could be dumped into the mixer giving, by this method, immediate control of the consistency of the concrete and the mixing operations.

Plotted in Fig. 4 are the slumps and Kelly Ball readings taken on this job. Twenty samples were taken and slumps made of each. As described above, these samples were from material tested for consistency by the ball.

Tests made on other jobs and in the laboratory have yielded a factor of 2 as the ratio of ball penetrations to slumps. In Fig. 4 that ratio is slightly less than 2, the average of the twenty ball penetrations being 1.32 in. The standard deviation is .446. Of the twenty slumps taken, the average is 2.5 with a standard deviation of .81. The concrete consistency as indicated by the slumps is not quite as uniform as that indicated by the Kelly Ball. Nine ball readings were about 1¼ in. Slumps taken of the same concrete ranged from 2 to 2¾ in. The spread, three-quarters of an inch of slump in nine samples, is as near perfect as slumps can be taken. The fact that the Kelly Ball reflected no differences in the consistency of the nine samples is not to its discredit. In each case shown the factor 2 could be used to translate Kelly Ball readings to slumps and the resulting approximate slump would be within the accuracy of the slump test. For example, there are four ball readings of 1 in.; using the factor 2, these readings would mean a slump of approximately 2 in. Actually the slumps are: 1.5, 1.7, 2, and 1.25.

These results check closely with slump cone-Kelly Ball comparisons established on other jobs. Some of these comparisons are reported in ASTM *Bulletin* No. 163 and

"Inspection of Ready-Mixed Concrete," ACI JOURNAL, June 1950. It is shown too that the Kelly Ball gives reliable results that can be used for concrete consistency control. Further, the speed and ease of testing with the Kelly Ball is a great advantage over the slump cone method of test. Again, the contractor is made happy because he is in no way delayed while the test is being made. We have noted that on jobs where the ball is used, contractor-inspector relationships are greatly improved. And not least of all advantages of the Kelly Ball is the ease of cleaning up.

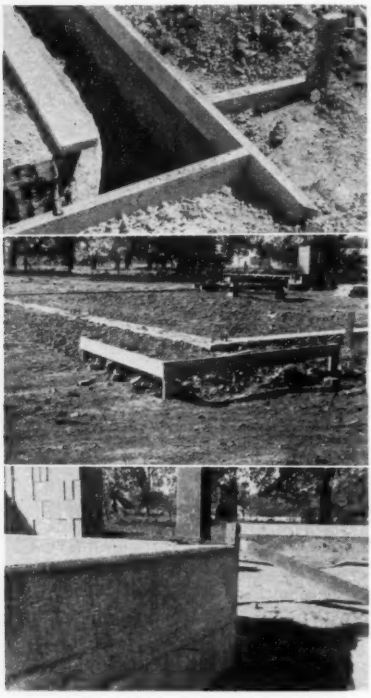
E. L. HOWARD and GEORGE LEAVITT, Pacific Coast Aggregates, Inc., San Francisco, Calif.

Anchor Bolts for Setting Column Bases (LR 48-17)

The accompanying figures show the application of leveling screws for setting column bases described earlier.* That this method permits rapid construction is illustrated by the fact that the first shovel of dirt was handled September 14 and the steel was erected and bolted on September 26, only 8 working days later.

*See ACI JOURNAL Sept. 1951, p. 93.

CHARLES MACKLIN, architect and structural engineer, Springfield, Ill.



Corner detail of outside footing form. Extending one 2 x 6 beyond the other increases stability of the corner and saves one saw cut. Stable soil served as below-grade form

Footing completed and back filled. Note 3- and 4-bolt anchor bolt groups.

Corner column in place on three anchor bolts. When steel is aligned column bases are grouted in place. Erection of curtain walls followed

Current Reviews*

of Significant Contributions in Foreign and Domestic Publications

Rational type of mushroom tank (Un tipo razionale di serbatoio a fungo)

SILVIO CERVI, *Giornale del Genio Civile* (Rome), V. 88, No. 7 & 8, July-Aug. 1950, pp. 464-467

Reviewed by GENNARO MIANULLI

Description of a mushroom-shaped tank designed by the author. Its merits are compared with the Itze type of tank.

Standard manufacturing processes for the cement industry (Sull' urgenza di generalizzare la razionalita'di fabbricazione di cementi)

FABIO FERRARI, *Il Cemento* (Milan), V. 47, No. 10, Oct. 1950, pp. 170-172

Reviewed by GENNARO MIANULLI

Report of the author's investigation of the manufacturing processes of cements in Italy and other countries. The main purpose of the report is to set cement production on a rational and scientific basis.

Ultimate strength theory for eccentrically loaded reinforced concrete columns

HENRY J. COWAN, *Magazine of Concrete Research* (London), No. 7, Aug. 1951, pp. 19-22

AUTHOR'S SUMMARY

Presents a method for computing the ultimate strength of reinforced concrete sections subject to combined bending and compression. It is based on the idealized stress-strain diagram proposed by Emperger and developed by Jensen, in which the compressive strain of the concrete at failure is divided into a perfectly elastic and a perfectly plastic stage. Equations for the ultimate column load are derived for primary tension failure of the steel and for primary compression failure of the concrete.

Flat slab design (Contributo al calcolo delle piastre di cemento armato rettangolari vincolate al contorno)

A. DANUSSO, *Il Cemento* (Milan), V. 47, No. 10, Oct. 1950, pp. 172-175.

Reviewed by GENNARO MIANULLI

Continuation of the eleventh installment on the investigation of flat plates supporting concentrated loads and with all four edges restrained. Simplifications are introduced for practical application.

School buildings of "Les Sablons" at Saint-Pierre-des-Corps. (Groupe scolaire des Sablons a Saint-Pierre-des-Corps)

CHARLES DORAIN and JEAN DORAIN, *Annales de L' Institut Technique du Batiment et des Travaux Publics* (Paris), Gros-Oeuvre, No. 1, No. 203, New Series, July-Aug. 1951

Reviewed by C. P. SIESS

Design and construction of school buildings featuring precast concrete beams and frame members. Treatment is primarily architectural; few engineering details are described.

A theory of deflection of reinforced concrete beams under short-term loads

F. A. BLAKEY, *Magazine of Concrete Research* (London), No. 7, Aug. 1951, pp. 3-8

AUTHOR'S SUMMARY

From elementary elastic theory a method of calculating the deflection of simply reinforced concrete beams under short-term loads is developed. It is shown that the results so obtained are in better agreement than those obtained by any other method of calculation.

*A part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 23, No. 4, Dec. 1951, *Proceedings* V. 48. Address 18263 W. McNichols Rd., Detroit 19, Mich. Copies of articles or books reviewed are not available through ACI. In most cases they can be obtained direct from the original publishers. Address, when available, will be furnished by ACI on request.

Tests on laterally loaded piles (Esperienze su pali infissi sollecitati normalmente all'asse)

CARLO RAYMONDI, *Giornale del Genio Civile* (Rome), V. 88, No. 8 & 9, July-Aug. 1950, pp. 428-441
Reviewed by GENNARO MIANULLI

Tests and observations were made on single reinforced concrete hollow piles, vertically driven into the ground, to ascertain behavior under transverse forces. Poor soil and dynamic loadings necessitated actual pile driving. Results are tabulated. All the experimental data are compared with analytical investigation.

Cross sections of prestressed beams obtained by direct methods (Sul calcolo diretto delle travi a sezione rettangolari ed a t in conglumerato precompresso)

UGO FLUXA, *Il Cemento* (Milan), V. 47, No. 10, Oct. 1950, pp. 175-178
Reviewed by GENNARO MIANULLI

A rapid method which can be directly applied for proportioning the cross section of rectangular and T-beams. Gujon's investigation is followed; diagrams are developed to simplify the investigation for the most economical dimensions for a prestressed beam.

Importance of the steel quality in reinforced concrete construction (L'Importanza delle qualita' dell'acciaio nelle costruzioni cementizie)

G. NEUMANN, *Il Cemento* (Milan), V. 47, No. 9, Sept. 1950, pp. 162-165
Reviewed by GENNARO MIANULLI

In this second installment, the author reviews steel manufacturing in Europe and discusses the physical and chemical properties and its use in reinforced concrete structures, the preference of one quality over the other, and high strength steel for prestressed structures.

Air-entraining concrete (in Czech)

O. VALENTA *Publications of the Faculty of Civil Engineering*, Czech Technical University (Prague), Reprint 2 1950
Reviewed by IVAN M. VIEST

In this up-to-date publication on air-entraining concrete, the author draws on American research and practice. Three chapters are devoted to the technological aspects and the controls of the air-entrainment, one chapter deals with U. S. specifications, and the specific needs for the use of air-entraining concrete in Czechoslovakia are discussed in one chapter.

Vibrated concrete

D. A. STEWART, *The Reinforced Concrete Review* (London), V. II, No. 5, Jan. 1951, pp. 277-314
Reviewed by C. P. SIESS

The results of research are reviewed, and after brief theoretical consideration of the phenomena of vibration the author discusses at length the actual phenomena observed in practise. Various types of vibrators, their characteristics, and the conditions for their most effective use are considered. The design of mixes for use with vibration and the relation between the vibrator characteristics and the properties of the mix are discussed from a practical point of view.

Problem of prestressing redundant structures (Il problema della precompressione su solidi vincolati iprstaticmente)

GIUSEPPE RAINERI, *Giornale del Genio Civile* (Rome), V. 88, No. 7 & 8, July-Aug. 1950, pp. 456-463
Reviewed by GENNARO MIANULLI

In the second installment, the selection between cable curve and pressure line is taken into consideration; also allowance is made for the condition which must satisfy the curve and pressure line (when this condition is possible), support reaction, the possibilities of elastic instability and the probable proportions for a redundant structure under dead and live loads.

Deflections in gridworks and slabs

WALTER W. EWELL, SHIGEO OKUBO and JOEL I. ABRAMS, *Proceedings*, ASCE, V. 77, Sept. 1951, Separate No. 89, 22 pp.
AUTHORS' SUMMARY

Because the exact determination of elastic deformations in plates and slabs by the mathematical theory of elasticity has proved to be impractical in all but the simple cases, engineers have been forced to devise approximate means of solution. A horizontal gridwork of beams intersecting at right angles, with deformation characteristics analogous to those of a slab, will develop an elastic surface under normal load similar to the surface of the slab under the same load. A technique is presented to determine the elastically deformed surface of a plate or slab that has been transformed into an equivalent grid system. The method employs an auxiliary force system for controlling vertical displacements of the joints and a moment and torque distribution process for transmission of the displacement effects.

Basic principles: Foundations of power hammers (in German)

C. Herrmann, *Schweizerische Bauzeitung* (Zurich), V. 69, No. 11, Mar. 1951, pp. 143-146
Applied Mechanics Reviews, Sept. 1951 (White)

Author analyzes problem, considering hammer, anvil, and foundation mass as rigid bodies coupled to each other. Impact of hammer on anvil is treated on basis of conservation of momentum and a coefficient of restitution. The anvil is assumed separated from the foundation by an elastic pad with damping. Author calculates maximum pressure on pad between anvil and foundation, maximum pressure of foundation on earth, and efficiency of hammer (proportion of useful energy to total energy).

Auxiliary vibrator for hand and machine sieving

T. H. Hughes, *The Engineer* (London), V. 192, No. 4985, Aug. 10, 1951, pp. 188-190
Reviewed by Aron L. Mirsky

Describes a simple 3-volt dc vibrator to be attached to a sieve or sieves during either hand or machine sieving. Test data are given showing that use of the vibrator increases materially the rate at which superfine particles, which normally tend to clog the sieve openings, especially in the case of "sticky" materials, pass through a fine sieve.

Although the vibrator was developed for use with a sticky coal, on which the tests were performed, it might be of considerable use in concrete laboratories in the opinion of the reviewer.

Computation of floor beams and influence on the distribution of concentrated loads to girders in a reinforced concrete bridge (Sul calcolo dei traversi e sulla loro distribuzione dei carichi concentrati fra le travi longitudinali dei ponti in cemento armato)

Ugo Fluxa, *Giornale del Genio Civile* (Rome), V. 88, No. 8 & 9, July-Aug. 1950, pp. 442-455
Reviewed by Gennaro Mianulli

The influence exerted on the distribution of load between secondary and main members in a reinforced concrete beam-girder type bridge is analyzed by an exact method based on the application of the general equation of continuity. The complexity involved in the problem is solved by approximations and simplifications. The results are compared with studies made previously by other authorities.

Concrete shell structures in the West

E. Kendall Thompson, *Architectural Record*, No. 32, Feb. 51, pp. 2, 3
Reviewed by J. J. Polivka

Short description and pictures of the first large reinforced concrete dome of thin section in the United States over the Second Church of Christ Scientist in Los Angeles constructed in 1909 and of several recently built concrete shell structures in the West; shell dome of San Francisco planetarium, 253-ft span barrel shell roof of Municipal Stadium in Denver, multiple barrel shell of Western Electric Co., Denver, and others, pointing to structural principals and advantages, especially to great economy. Concrete shell thickness down to ¾ and 1¼ in. are not unusual.

Elementary lessons in elasticity with applications in structural design (in Spanish)

E. Torroja, Editorial Dossat, S.A., Madrid, 1951, 2nd Edition, 310 pp.
Reviewed by J. J. Polivka

A basic textbook on structural applications of the theory of elasticity in Spain and Latin American countries written by one of the leading structural engineers who is internationally known in connection with unusual shell-type structures and long-span arch bridges in reinforced concrete. In the first part general problems of three-dimensional stress analysis are presented, the second part contains two-dimensional stress problems and the third part is devoted to structural problems frequently encountered in engineering practice. Methods of experimental stress analysis, especially photoelasticity, are presented in condensed form.

Strength of reinforced concrete beams in shear

C. B. Wilby, *Magazine of Concrete Research* (London), No. 7, Aug. 1951, pp. 23-30
From Author's Summary

Deals mainly with the effect of the following factors on the ultimate shearing strengths of beams; percentage and spacing of web reinforcement for stirrups inclined at 45° to the horizontal, ratio of span under shear to the effective depth for beams with plain webs and for two series of beams with different web ratios, and the percentages of longitudinal compression and tension reinforcement for beams with plain webs and for two series of beams with different web ratios.

In the beams with reinforced webs the stirrups were inclined at 45° to the horizontal, and a total of 79 shear failures were investigated from a study of asymmetrically reinforced beams and eccentrically loaded beams.

The basic assumptions of the various theories in existence were investigated by various measurements of concrete and steel strains, deflections, and widths of cracks.

The results are discussed with regard to existing theories and past research work. It is shown that even small percentages of web reinforcement were effective in increasing the ultimate shearing failing load. The yield stress of the stirrup was never exceeded and the internal and external shearing forces can only be balanced by assuming that the beam behaves like an open-frame girder.

How temperature and moisture changes may affect the durability of concrete

S. L. MEYERS, *Rock Products*, V. 54, No 8, Aug. 1951, pp. 153-157 and 178

Reviewed by DONALD M. AGRIMSON

Excluding heat of hydration in recently placed concrete, the thermal effects of temperature differences in concrete may be divided into four possible classifications.

(1) The effect of changes on the aggregates alone, many being nonuniform in composition, or made up of dissimilar crystals, as a granite, where each kind of crystal may have a different rate of thermal expansion.

(2) The effect of changes on the cement paste; the magnitude of expansions of hardened cement paste varies with relative humidity and age.

(3) The expansion of the mortar-cement matrix (paste plus sand or fine aggregate) and the stress set up at the contact of paste and fine aggregate due to differences in thermal coefficients of the two materials.

(4) The expansion of the concrete, considering the concrete to be made up of coarse aggregate and mortar (assuming the mortar to expand as a one-component material) and the thermal stress set up at the contact of mortar and coarse aggregate. The coarse aggregate-mortar bond appears to be very important in resisting stress, due to thermal dissimilarities between the two materials.

The reader will find in this article a wealth of information on the points listed. Supplementing the discussion are several tables and graphs illustrating the findings of various tests conducted relative to the properties of aggregates, cement paste, cement-sand mortars, and concrete.

Construction of reservoirs for hydrocarbons in reinforced concrete made tight by means of a hydraulic partition (La construction de reservoirs a hydrocarbures en beton arme etanches au moyen d'une paroi hydraulique)

L. P. BRICE, *Le Genie Civil* (Paris), V. 128, No. 13, July 1, 1951, pp. 247-249

Reviewed by ARON L. MIRSKY

An article summarizing some twelve years' experience with reinforced concrete tanks for gasoline storage. The author ascribes the permeability of concrete to gasoline, as compared to that for water, to the differences of viscosity and surface tension of the two liquids; should gasoline be on one side of a concrete wall, and water on the other, with the pressure of the water equal to or greater than that of the gasoline, the wall would be rendered impervious to the gasoline. This principle is applied to the construction of the storage tanks, which consist in effect of two concrete tanks one inside the other, with the annular space (formed of hollow block or similar material) being filled with water. The two tanks are rigidly tied together by reinforcement passing through the annular space.

Among the advantages of this type of construction is the low thermal conductivity of the composite wall.

The law of grading for concrete aggregates

L. BOYD MERCER, Melbourne Technical College Press, 1951

Reviewed by W. H. PRICE

The investigation of 28 different aggregate gradings, some of them containing as many as 4 gaps, in 36 concrete mixes in the laboratory are described in considerable detail. All mixes contained 6 sacks of portland cement per cubic yard of concrete. The water-cement ratio of 28 mixes was maintained at 0.6 by weight, of 6 mixes at 0.5 by weight, of 2 mixes at 0.7 by weight. The slump of the concrete was allowed to vary from 0 to 8 in., depending upon the water-cement ratio and particular aggregate grading used.

It was concluded from these tests that aggregate gradings are important only insofar as they affect segregation and placeability

of the fresh concrete, and that where the concrete is thoroughly compacted the grading of the aggregate has no influence on the compressive strength and other desirable properties of the hardened concrete. Aggregates having discontinuous or gap gradings can present superior placeability and behavior to those having continuous straight-line gradings, and it is recommended that the gradings of the available aggregates be investigated in trial mixes with consideration being given to the handling and placing conditions on the job before requiring expensive processing to obtain some continuous grading. It is indicated from the tests that gaps in the aggregate grading with sufficient fine material preceding the gap to hold the cement paste and sufficient voids in the material following the gap to prevent particle interference will improve the workability of the concrete as compared to concretes made with continuous gradings. It is also indicated that for concrete containing 1½-in. maximum size aggregate that a gap in the ⅜ in. to No. 4 size is advantageous.

Results of tests on prestressed concrete members (Quelques experiences et realisations—Le beton precontraint)

La Technique Moderne-Construction (Paris), V. 6, No. 6, June 1951, pp. 224-226
Reviewed by ALEXANDER TURITZIN

A summary of tests made by Professor Campus of the University of Liege, Belgium, is given. These tests consisted of measuring flexural deformation of concrete and reinforcement under uniformly increasing load. Tests were carried to destruction. The SR-4 strain gages and the various fleximeters and deformeters were placed at several points and the load was gradually applied by means of jacks. Four different types of prestressed concrete beams were tested for flexural stresses; inverted T- and U-beams, rectangular beams and cylindrical beams.

Test results showed that in the prestressed concrete beams, with steel wires bonded to concrete, the deformation was proportional to the applied load up to the point of the appearance of the cracks. In beams with steel wires isolated from the concrete, the deformation was not always proportional to applied load, probably due to the friction of the wires and the sheathing.

In addition to the above mentioned tests, prestressed concrete railroad ties were tested for fatigue and prestressed concrete pipes were tested for axial compression and longitudinal flexure. In all of the tests performed on the railroad ties, the limit of endurance of the concrete was not reached; the failure occurred either in bond or by rupture of reinforcing wires. The prestressed concrete pipes were found to be almost 60 percent stronger than the non-prestressed reinforced concrete pipes.

In conclusion, the author states that the prestressed concrete members tested to destruction showed an elastic behavior up to the point of the appearance of cracks and also showed a high resistance to fatigue. The coefficient of safety of the prestressed concrete members is different from the coefficient of safety of the non-prestressed members and therefore when applied to the prestressed concrete members our notion of safety must be altered.

Concrete roads repaired by application of a new surface course (Herstel van cementbetonverhardingen)

A. K. W. UNGER and W. BARENTZEN, *Cement* (Amsterdam), No. 5-6, 1951, pp. 98-100
Reviewed by J. W. T. VAN ERP

Most of the early skepticism about concrete roads was on account of the presumed difficulties in repairing them. Particularly uneven settlement of adjoining sections were considered impossible to repair except by covering them with heavy layers of bituminous materials or by the expensive operation of jacking up the settled slabs, this last method proving even more expensive on account of fractures leading to early deterioration under traffic. In later years, however, successful repairs have been made by the application of new surface courses and it is worth while to note the procedure found to be the best.

The to be repaired concrete roads were approximately 20 years old. They showed uneven settlement as well as cracks and had no superelevation in curves. The most successful method of repair was simple but called for rather painstaking execution and careful supervision. In general where the new top layer was less than ¾ in. thick, the old concrete slab surface was roughened or sometimes holes were drilled near the edge of the old slab to insure adequate bond-

ing of the new concrete to the old. Then the roughened as well as the rest of the old slab was thoroughly brushed with diluted hydrochloric acid (1 part concentrated acid in 1 part water). After five minutes the surface was washed clean. On this moist surface a coating about 1 mm thick of a cement and water paste was brushed in. The water contained ½ lb of dehydrated calcium chloride per gallon to accelerate concrete set. The concrete surface course applied consisted of 1 part cement and 3 parts sand for thin layers and 1 part cement, 2 parts sand and 3 parts gravel for heavier layers. Calcium chloride in the same amount as above was used. The surface course was well tamped and broomed.

The actual costs are of little interest because of the low European labor/material cost ratio, but the relative cost of the repairs as compared to a complete renewal of the concrete slab should prove of some value. The cost ratio of the repair job including labor, material and equipment rental against complete renewal of the old concrete slab was .325 for ¾-in. layers, .245 for layers up to 1¼-in. and .660 for layers up to 3-in. thick, such as needed to add superelevation for curves.

The cost of the second and third are lower due to the omission of roughening the old slab. The cost ratio for the first item, therefore, should be higher for conditions in this country.

Further notes on the principles and design of prestressed concrete

P. W. ABELES, *Civil Engineering and Public Works Review* (London), Parts 1-10

Reviewed by CHARLES W. DOHN

This series is an extension of an original discussion by this well-known author which appeared in the January through October, 1948, issues of the same publication. Results of subsequent investigations lead to comment clarifying the behavior of prestressed concrete particularly with respect to cracking and failure. Required factor of safety and principles of design of prestressed concrete for different cases are discussed. The articles are illustrated with load deflection curves of prestressed concrete beams and they contain an extensive bibliography. The author notes the extraordinary resilience of prestressed beams even after being loaded to 96.4 percent of ultimate strength. The author recommends a factor of safety of two for under-reinforced prestressed concrete beam design and a factor of safety up to three for over-reinforced prestressed concrete beam design. This is because of the certainty with which the minimum strength values of steel can be guaranteed as contrasted to the uncertainty of minimum strength values which can be guaranteed for concrete. The author discusses "balanced" design and the avoidance of a design causing danger of sudden failure due to over-reinforcement. Diagrams are given for plastic behavior of under- and over-reinforced concrete beams. Four types of structure defined varied from over-prestressed to under-prestressed or non-prestressed and point is made that the two intermediate classes are best from point of view of absorption of shock and cracking and deflection. A high degree of safety against cracking cannot be combined with the requirements of a structure giving an early warning of overloading, having a greater resistance to shock and being more economical. Undue preference should not be given for security against cracking in prestressed concrete. Hair cracks under rare loading would be of no harm, provided that the closing of these cracks is ensured under ordinary loading.

Part 1, V. 45, No. 529, July 1950, pp. 443-445

Considers the number of recent tests carried out on prestressed concrete sections and outlines their effect on present design methods.

Part 2, V. 45, No. 530, Aug. 1950, pp. 508-512

Considers over-reinforced and under-reinforced beams, factor of safety and load factor.

Part 3, V. 45, No. 531, Sept. 1950, pp. 579-581

Investigates the present state of knowledge of the resistance moment and the effect of loads due to self weight and to handling.

Part 4, V. 45, No. 532, Oct. 1950, pp. 657-662

Discusses some aspects of transformation load, factor of safety, discontinuity of properties, and partially prestressed structures, and also identifies four main types of prestressed structure.

Part 5, V. 45, No. 533, Nov. 1950, pp. 728-731

Compares the intermediate boundaries and the characteristics of the four main

types of structures, and deals with the co-operation of bonded, tensioned and untensioned wires.

Part 6, V. 46, No. 535, Jan. 1951, pp. 38-40

Investigates the problem of cracking in reinforced and prestressed concrete and examines the suitability of cross section.

Part 7, V. 46, No. 537, Mar. 1951, pp. 187-190

Gives an example of the design of a box-shaped highway bridge deck, and considers the minimum depth of the compressive flange of I-shaped or box girders.

Part 8, V. 46, No. 538, Apr. 1951, pp. 262-264

Considers the minimum depth of the compressive flange of I-shaped and box girders, the deflection of prestressed concrete members, the bending up of post-tensioned cables in beams in which the self weight always counteracts, etc.

Part 9, V. 46, No. 540, June 1951, pp. 439-442

Gives an example of another method of treating asymmetrical sections, and considers the bending-up of cables in beams in which the self-weight counteracts.

Part 10, V. 46, No. 541, July 1951, pp. 524-

Considers in detail the shear reinforcement requirements of prestressed concrete design n the light of some recent experiments.

PERIODICALS REGULARLY SCANNED FOR REVIEWS

The many requests for information concerning the source material of reviews appearing in the "Current Reviews" section indicate the desirability of publishing a list of those periodicals being regularly scanned for articles dealing with the field of concrete. Included also are the publishers' addresses for convenience should correspondence with them be desired.

Annales de L'Institut Technique du Batiment et des Travaux	28 Boulevard Raspail, Paris 7, France
Bauingenieur	Reichpietschufer, 20, Berlin, W35, Germany
Bautecknik, Die	Hohenzollerndam 169, Berlin-Wilmersdorf, Germany
Betong	Odengatan 41, Stockholm, Sweden
Beton og Jernbeton	Laboratoriet for Byningsteknik, Oster Volgade 10C, Copenhagen K, Denmark
Beton-und Stahlbetonbau	Hohenzollerndamm, 169, Berlin-Wilmersdorf, Germany
Beton-Teknik	Kalvebod Brygge 2, Copenhagen V, Denmark
Betonstein Zeitung	Frankfurter Strasse 5, Wiesbaden, Frankfurt Am Main, Germany
Bulletin, ASTM	1916 Race St., Philadelphia 3, Pa.
Bulletin, Institution of Civil Engineers in Ireland	35 Dawson St., Dublin, Ireland
Bulletin, University of Illinois	University of Illinois Engineering Experiment Station, 106 Engineering Hall, Urbana, Ill.
Bulletin Technique de la Suisse Romande	Case Postale Riponne, 21 Lausanne, Switzerland
Cahiers du Centre Scientifique et Technique du Batiment	33 Rue Jean-Goujon, Paris 8, France
Cement	Sarphatistraat 54, Amsterdam C, Netherlands
Cement and Lime Manufacture	14 Dartmouth St., London (Westminster S.W.1), England

Cement och Betong	Malmo, Sweden
Cemento, Il	9 Via Settembrini, Milan, Italy
Cemento Portland	Calle San Martin 1137, Buenos Aires, Argentina
Civil Engineer Corps Bulletin	Bureau of Yards and Docks, U.S.N., Washington 25, D. C.
Civil Engineering	33 W. 39th St., New York 18, N. Y.
Civil Engineering and Public Works Review	154 Fleet St., London E.C. 4, England
Commonwealth Engineer, The	349 Collins St., Melbourne, C. 1, Australia
Concrete	400 W. Madison St., Chicago 6, Ill.
Concrete and Constructional Engineering	14 Dartmouth St., Westminster S.W.1, England
Constructional Review	14 Spring St., Sidney, N.S.W., Australia
Crushed Stone Journal, The	1415 Elliot Place, N.W., Washington 7, D. C.
Current Road Problems	Highway Research Board, 2101 Constitution Ave., Washington 25, D. C.
Deutscher Beton-Verein	Wisbaden, Germany
Digest	Building Research Station, Garston, Watford Herts, England
Engineering Experiment Station News	Ohio State University, Columbus, Ohio
Engineering News-Record	330 W. 42nd St., New York 18, N. Y.
Giornale del Genio Civile	Via Nomentana 2, Rome, Italy
Hormigon Elastico	Lavalle 1171, 5°, Buenos Aires, Argentina
Handlingar (Proceedings)	Ingeniors Vetenskaps, Akademien, Stockholm, Sweden
Hungarian Technical Abstracts	Sztálin-Tér 4, Budapest 2, Hungary
Ingenioren	Copenhagen, Denmark
Journal of the American Ceramic Society	2525 N. High St., Columbus 2, Ohio
Journal of the Institution of Engineers Australia	Gloucester & Essex Sts., Sydney, NSW, Australia
Journal of Research	National Bureau of Standards, Washington 25, D. C.
Magazine of Concrete Research	52 Grosvenor Gardens, London S.W.1, England
Osterreichische Bauzeitschrift	Mölkerbastei 5, Vienna 1, Austria
Polytechnisch Tydschrift	28, Balistraat, The Hague, Netherlands
Proceedings, ASCE	33 W. 39th St., New York 18, N. Y.
Public Roads	Bureau of Public Roads, U. S. Dept. of Commerce, Washington 25, D. C.
Reinforced Concrete Review, The	94-98 Petty France, London S.W.1, England
Revista de Obras Sanitarias de la Nation	Charcas 1840, Buenos Aires, Argentina
Revue des Materiaux	19 Rue Lafayette, Paris 9, France
Rock Products	309 W. Jackson Blvd., Chicago 6, Ill.
Structural Engineer, The	2 Bream Bldgs., London E.C.4, England
Technique Moderne-Construction, La	92 Rue Bonaparte, Paris 6, France
Teknisk Tidskrift	Kungsgatan 37, Stockholm C, Sweden
Teknisk Ukeblad	Oslo, Norway
Travaux	6 Ave. Pierre 1st de Serbie, Paris 16, France
Zement-Kalk-Gips	Biebricher Strasse, 74, Wiesbaden, Germany